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Critical Masses of Cylinders of Plutonium Diluted with Other Metals*

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Critical masses of 6-in. diameter cylinders of plutonium diluted in various volume ratios with steel, aluminum, thorium, air, and depleted uranium were determined. The cores were in 0, 2, 4.5, or 7.5-in. thick uranium reflectors and in 2, 4.5, or 7.5-in. thick thorium reflectors. From these data, the equivalent spherical critical masses of plutonium have been estimated. An appendix gives reflector savings of undiluted plutonium for a variety of materials.

PART I. URANIUM-REFLECTED CYLINDERS

This series of sixty measurements established the critical masses of 6-in. diameter cylinders of plutonium diluted in various volume ratios with steel, aluminum, thorium, and depleted uranium. Diluted plutonium cores were unreflected or in 2, 4.5, or 7.5-in. thick uranium reflectors. The resulting data are intended to help establish the effects of representative structual and fertile materials on the size of plutonium-fueled fast reactors. This is to be accomplished most generally by using the resulting critical dimensions for testing the accuracy of appropriate S_n multigroup calculations (*1*).

Materials

The fissionable material available consisted of thirty-one delta-phase (low-density) plutonium plates sealed in thin nickel cans. The average dimensions of the uncoated plutonium were 5.934 in. diameter by 0.123 in. thick, and with nickel coating they were 5.967 by 0.135 in. The Pu mass of each plate was ~0.9 kg (~95% Pu^{239}, ~5% Pu^{240} with traces of Pu^{241} and Pu^{242}).

The diluent plates consisted of type 304 stainless steel, 1100F aluminum, thorium, and depleted uranium. The average dimensions of these plates were 5.967 in. diameter by $\frac{1}{8}$, $\frac{1}{4}$, or $\frac{1}{2}$ in. thick.

The 2- and 4.5-in. thick depleted uranium reflectors were built up of concentric cylinders in 10, 8, 4, 2, 1, and $\frac{1}{2}$-in. heights with appropriate disks to seal the ends. Natural uranium rings 0.6 in. in height fitted about the 4.5-in. thick reflector to form the 7.5-in. thick reflector wall.

Assembly Machine

The Comet universal assembly machine located in one of the Pajarito laboratories for remote operation was used for these measurements. This machine consists of a hydraulic lift directly beneath a stationary steel platform. For the unreflected systems (Fig. 1), a 0.015-in. thick stainless steel diaphragm was used to support the upper portion of the material. For the reflected systems, however, a heavy-duty steel platform supported the peripheral reflector so that the core (with end-reflector pieces) could be raised into it (Fig. 2).

Procedure

In the unreflected case, half of a known-safe Pu assembly (with diluent) was stacked on the thin diaphragm and the other half on a light Al support (to minimize incidental reflection) on the lift. The lower portion of the assembly was then raised remotely until it contacted the diaphragm. Responses of two BF_3 counters in polyethylene geometries to the neutron leakage from the Pu cylinders were the primary data obtained. No neutron source was used other than the natural emission from the Pu plates.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

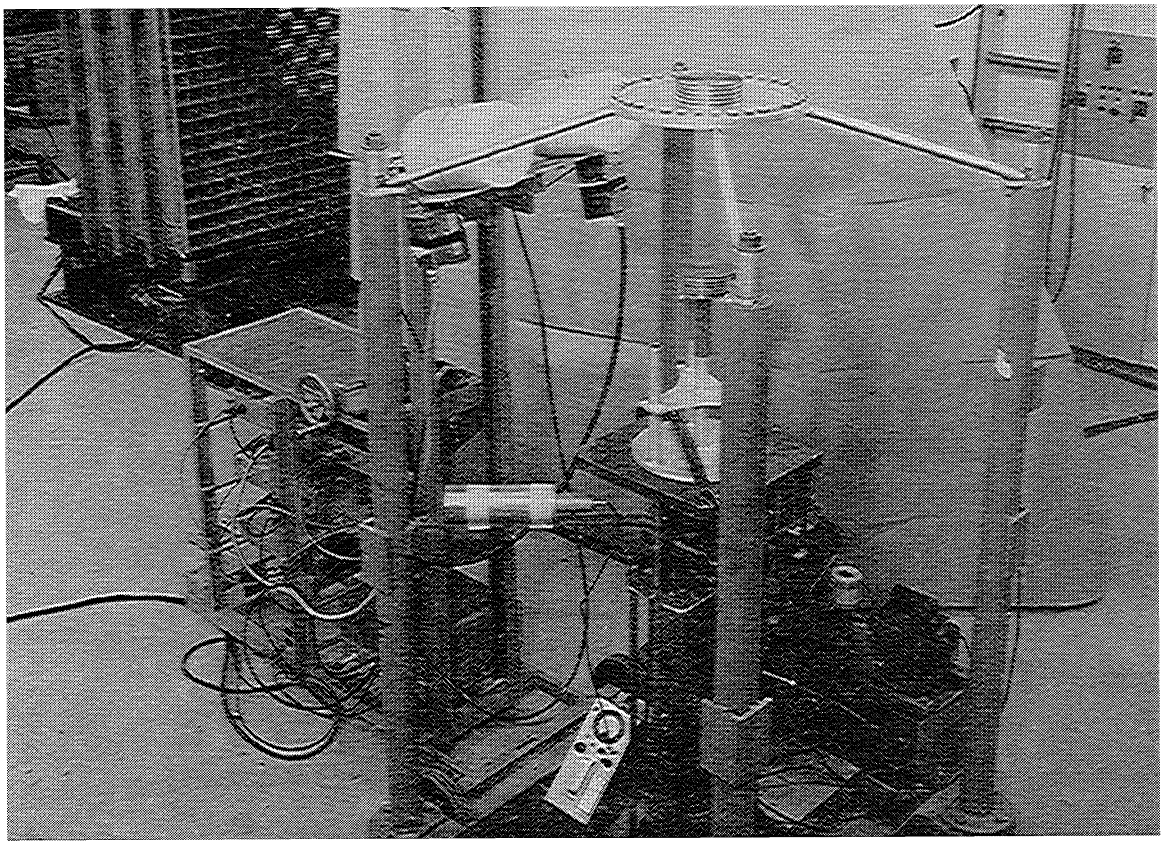

FIG. 1. Setup for measurements of unreflected dilute Pu cylinders on Comet universal assembly machine.

After disassembly, Pu and diluent materials were added, and the procedure repeated until a safety limit corresponding to a multiplication of 100 was approached or the material was exhausted. Where high multiplication was attainable, measurements such as this permitted extrapolation to the delayed critical configuration. In addition, end reflector savings were determined by placing $\frac{1}{2}$-in. thick disks of various materials on top of the otherwise unreflected Pu cylinder. Results, expressed in cm of plutonium, are included as an appendix to this paper.

For the reflected systems, the lateral uranium reflector was seated on the steel platform (reflector height was adjusted with height of the Pu cylinder). The Pu and diluent disks were assembled on the lift with proper thickness of reflector on top and bottom and encased in a 0.030-in. thick steel guide sleeve. This arrangement is illustrated in Figs. 2 and 3. The core was raised remotely into the reflector annulus for a series of neutron counts as described in the unreflected case.

RESULTS

Curves were plotted based upon the counting rates recorded for each configuration, and extrapolated critical mass values determined. Pu mass (a quantity proportional to the neutron source strength) divided by counting rate less background vs plutonium mass was always plotted in course of the experiment where reasonably high counting rates were attainable (Fig. 4). For those dilute systems where high counting rates were unattainable, it was more convenient to use reciprocal (height plus extrapolation length) squared as the abscissa so as to obtain approximately linear plots as illustrated in Fig. 5. Tables I through IV summarize the critical cylinder specifications.

ESTIMATION OF SPHERICAL CRITICAL MASSES

In order to see qualitatively the dependence of critical size on material composition, subject to fixed geometrical shape, we have included in Tables I–IV an estimate of the spherical critical mass correspond-

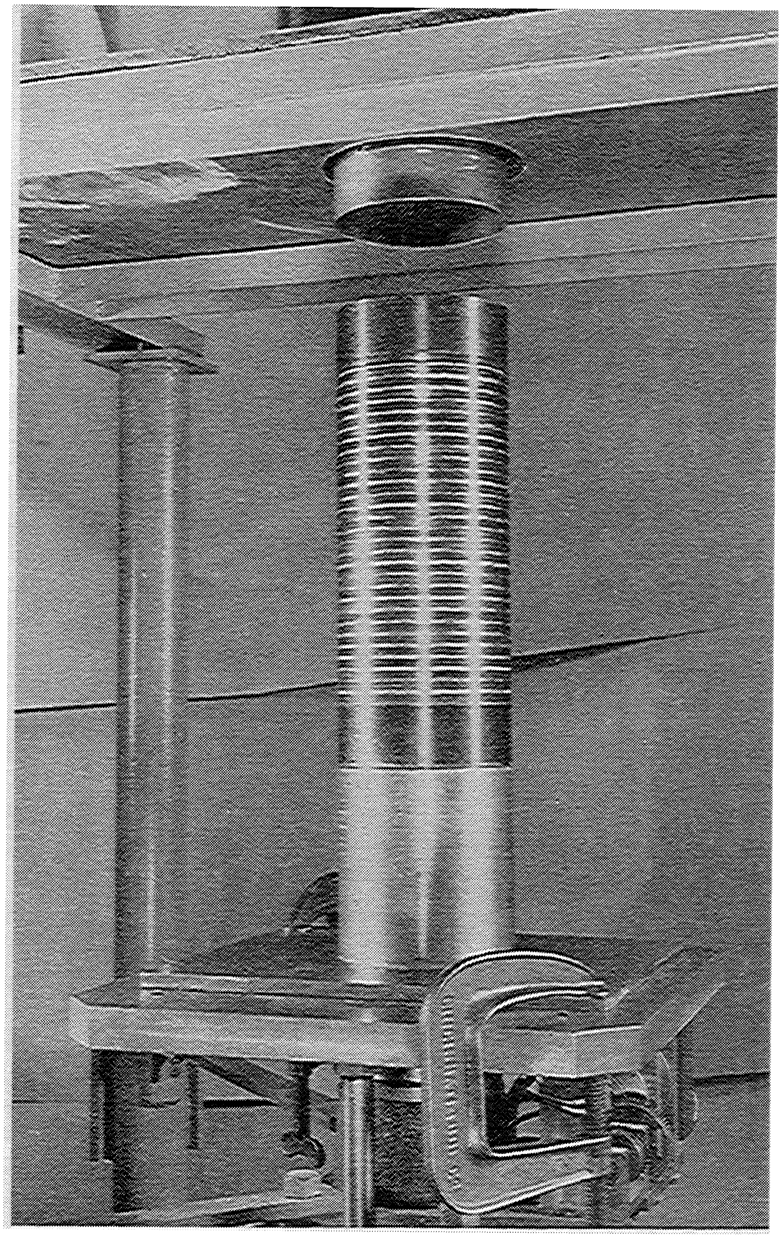

FIG. 2. Dilute Pu cylinder (Pu:U^{238} = 1:2) stacked on lift with 2-in. thick reflector cylinders on top and bottom of core.

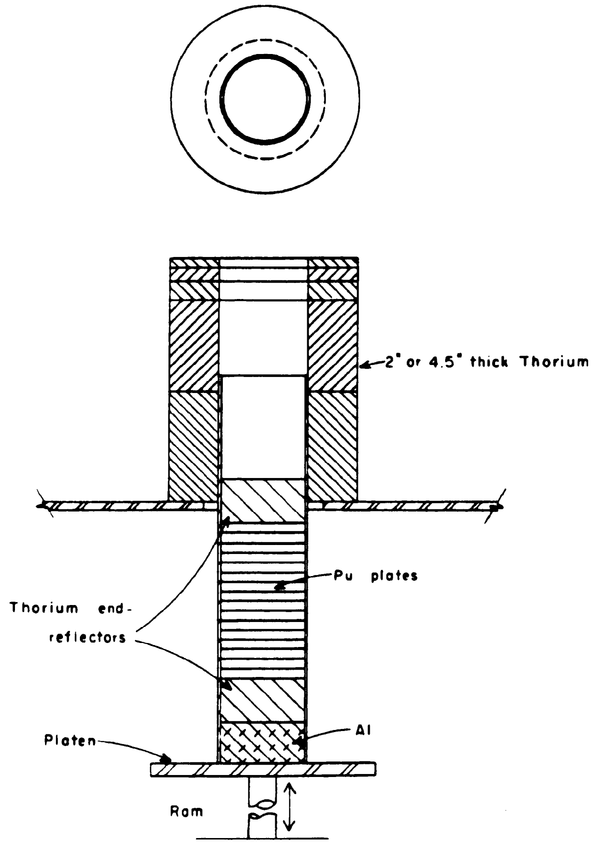

FIG. 3. Setup for reflected measurements of dilute Pu cylinders on assembly machine.

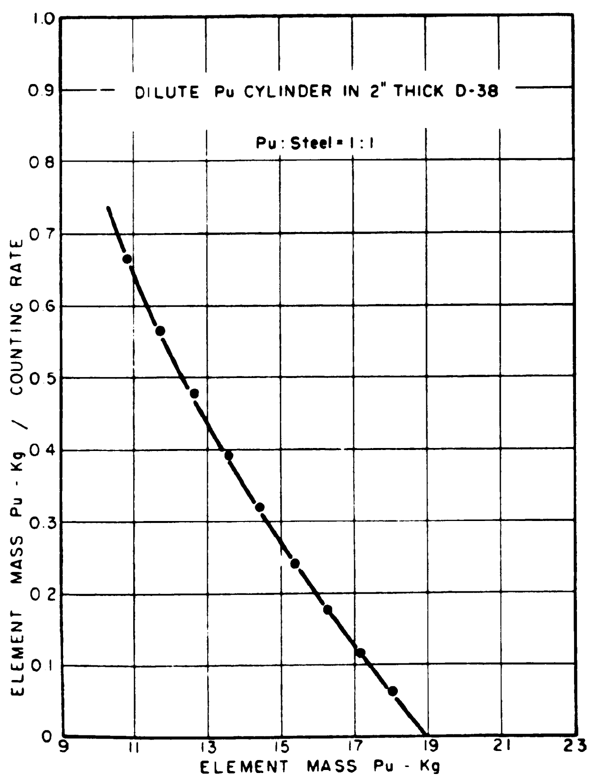

FIG. 4. Pu mass divided by counting rate versus Pu mass for volume ratio Pu:Steel = 1:1 in 2-in. thick uranium.

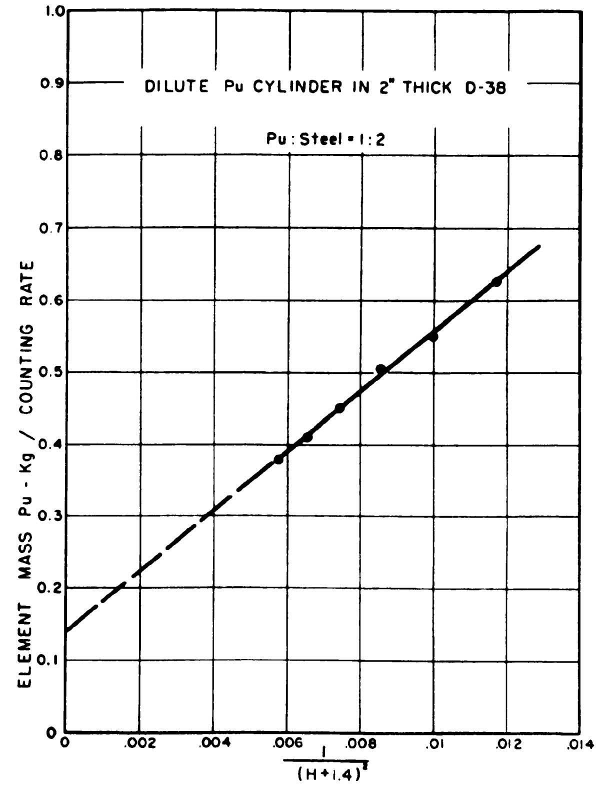

FIG. 5. Pu mass divided by counting rate versus square of reciprocal (height plus extrapolation length) for volume ratio Pu:Steel = 1:2 in 2-in. thick uranium.

ing to each cylinder measurement. These estimates are based on the buckling relation

$$\left(\frac{\pi}{R + \lambda_s}\right)^2 = \left(\frac{2.405}{r + \lambda_c}\right)^2 + \left(\frac{\pi}{h + 2\lambda_c}\right)^2$$

which relates critical dimensions of spheres and cylinders of the same composition. Here R is the critical sphere radius, r and h the observed critical cylinder radius and height, and λ_s and λ_c are effective sphere and cylinder extrapolation lengths.

The extrapolation lengths were estimated in the following empirical fashion: (1) The value $\lambda_s(x, t)$ for a core of material x reflected by a thickness t of uranium was taken equal to

$$\lambda_s(\mathrm{Pu}, t)[(1 + f_{\mathrm{Pu}})\Sigma(\mathrm{Pu})]/[(1 + f_x)\Sigma(x)]$$

where the one group parameters f (the excess number of neutrons produced per collision) and Σ (the macroscopic transport cross section) were obtained from the one-group cross sections listed in the accompanying paper by Engle *et al.* (*2*); the extrapolation length $\lambda_s(\mathrm{Pu}, t)$ was obtained from critical mass data on uranium-reflected plutonium spheres (*3*) using the computed value

$$\lambda_s(\mathrm{Pu}, 0) = 0.71/(1 + f_{\mathrm{Pu}})\Sigma(\mathrm{Pu}) = 2.15 \text{ cm}.$$

TABLE I

Critical Parameters of Unreflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
Diluent[a]	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Core-average densities (g/cc) Pu	Core-average densities (g/cc) principal diluent	Core-average densities (g/cc) Ni	Estimated spherical critical mass-kg Pu
None	--	20.24	21.39	3.23	14.27	--	0.651	17.5
Depleted uranium	2:1	25.24	27.3	6.066	9.83	5.97	0.448	24.7
Steel (SS 304)	2:1	27.06	32.8	7.32	9.78	2.50	0.446	28.6
Th	2:1	27.06	35.2	7.85	9.78	3.62	0.446	30.1
Al (1100F)	2:1	27.06	37.0	8.33	9.69	0.83	0.441	31.1
Air	2:1	27.06	77.1	16.85	9.98	--	0.455	43.7
Depleted uranium	1:1	27.06	<———— Not critical ————>					
Steel	1:1	27.95	<———— Not critical ————>					

[a] Depleted uranium in cores contains 0.28% U^{235}; the average content of reflector parts is not known.

TABLE II

Critical Parameters of 2-in. Thick Uranium Reflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Core-average densities (g/cc) Pu	Core-average densities (g/cc) principal diluent	Core-average densities (g/cc) Ni	Estimated spherical critical mass-kg Pu
None	--	10.73	11.15	1.715	14.18	--	0.652	8.5
Depleted uranium	2:1	12.53	13.00	2.919	9.71	5.95	0.446	11.6
Steel (SS 304)	2:1	13.92	14.10	3.155	9.75	2.51	0.446	12.9
Th	2:1	14.38	14.70	3.291	9.74	3.63	0.446	13.6
Al (1100F)	2:1	14.15	14.38	3.227	9.72	0.839	0.446	13.5
Air	2:1	14.42	15.05	3.29	9.97	--	0.455	14.6
Depleted uranium	1:1	15.39	15.60	4.561	7.46	9.03	0.339	15.0
Steel	1:1	18.72	19.00	5.580	7.43	3.78	0.338	18.4
Th	1:1	19.84	20.65	6.017	7.49	5.55	0.341	20.1
Al (1100F)	1:1	19.63	19.87	5.784	7.49	1.28	0.341	19.5
Air	1:1	21.66	22.38	6.43	7.60	--	0.346	22.3
Depleted uranium	1:2	27.06	28.92	12.491	5.05	12.22	0.230	21.7
Steel	1:2	27.03	<———— Not critical ————>					
Depleted uranium	1:3	27.03	<———— Not critical ————>					

TABLE III

Critical Parameters of 4.5-in. Thick Uranium Reflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Core-average densities (g/cc) Pu	Core-average densities (g/cc) principal diluent	Core-average densities (g/cc) Ni	Estimated spherical critical mass-kg Pu
None	--	8.99	9.28	1.419	14.26	--	0.652	7.1
Depleted uranium	2:1	8.99	10.62	2.40	9.65	5.88	0.441	9.1
Steel (SS 304)	2:1	10.57	11.22	2.59	9.46	2.48	0.441	9.9
Th	2:1	10.76	11.70	2.62	9.75	3.65	0.447	10.5
Al (1100F)	2:1	10.76	11.55	2.58	9.75	0.841	0.447	10.5
Air	2:1	10.83	11.79	2.59	9.95	--	0.453	11.1
Depleted uranium	1:1	12.55	12.65	3.72	7.42	9.08	0.340	11.8
Steel	1:1	14.35	14.48	4.26	7.42	3.81	0.340	13.9
Th	1:1	15.24	15.38	4.51	7.44	5.55	0.341	15.1
Al (1100F)	1:1	14.35	14.77	4.34	7.42	0.761	0.340	14.7
Air	1:1	15.38	15.67	4.46	7.65	--	0.348	15.9
Depleted uranium	1:2	17.99	18.55	7.99	5.06	12.28	0.231	16.7
Steel	1:2	24.31	25.32	10.97	5.04	5.14	0.230	21.1
Th	1:2	27.95	29.97	12.90	5.07	7.70	0.231	24.1
Al (1100F)	1:2	26.13	26.42	11.42	5.05	1.75	0.230	22.4
Air	1:2	27.03	30.10	12.84	5.11	--	0.233	28.3
Depleted uranium	1:3	27.16	107.9	61.8	3.81	13.87	0.174	25.7
Steel (SS 304)	1:3	28.05	<—— Not critical ——>					

TABLE IV

Critical Parameters of 7.5-in. Thick Uranium Reflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
					Core-average densities (g/cc)			
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Pu	principal diluent	Ni	Estimated spherical critical mass-kg Pu
None	--	8.04	8.96	1.37	14.23	--	0.652	6.8
Depleted uranium	2:1	10.32	10.35	2.31	9.77	5.94	0.447	8.8
Steel (SS 304)	2:1	10.76	10.88	2.43	9.76	2.51	0.447	9.5
Th	2:1	10.76	11.13	2.49	9.76	3.63	0.447	9.9
Al (1100F)	2:1	10.76	11.04	2.47	9.76	0.840	0.447	9.9
Air	2:1	10.80	11.25	2.47	9.92	--	0.453	10.4
Depleted uranium	1:1	11.73	11.96	3.51	7.44	9.05	0.339	11.0
Steel (SS 304)	1:1	13.33	13.55	3.97	7.45	3.80	0.339	12.7
Th	1:1	14.19	14.25	4.14	7.51	5.57	0.343	13.7
Al (1100F)	1:1	13.54	13.70	3.98	7.50	1.28	0.342	13.2
Air	1:1	14.41	14.58	4.18	7.62	--	0.348	14.6
Depleted uranium	1:2	16.66	16.91	7.29	5.06	12.27	0.231	15.5
Steel (SS 304)	1:2	21.67	21.78	9.49	5.01	5.11	0.228	19.3
Th	1:2	25.04	25.15	10.83	5.07	7.49	0.231	21.7
Al (1100F)	1:2	22.35	22.36	9.65	5.05	1.75	0.230	20.2
Air	1:2	24.37	24.85	10.58	5.12	--	0.233	24.9
Depleted uranium	1:3	27.03	30.13	17.24	3.81	13.87	0.174	19.2
Steel (SS 304)	1:3	27.03	← Not critical →					

(2) The value $\lambda_c(x, t, h/d)$ of a cylindrical core of material x, height to diameter ratio h/d, reflected by a thickness t of uranium was taken equal to

$$\lambda_s(x, t)[\lambda_c(U^{235}, t, h/d)/\lambda_s(U^{235}, t)]$$

where the bracketed term was evaluated from published U^{235} shape factor data (4).

There are a limited number of experimental checks on the accuracy of this shape conversion recipe for Pu cylinders (4) (see the tabulation).

Natural uranium thickness (in.)	Estimated critical mass, m_c (t), from cylinder data (kg)	Critical mass from sphere data (kg)
0	17.5	18.6
2	8.5	8.8
4.5	7.1	7.0
7.5	6.8	6.5

PART II. THORIUM-REFLECTED CYLINDERS

Critical mass measurements of Part I were repeated insofar as possible with the diluted Pu cores reflected by 2, 4.5, and 7.5-in. thick thorium. The fissionable material, diluent plates, assembly machine, and procedure were as previously described. The thorium reflector consisted of concentric cylinders that duplicated the uranium reflector pieces, the only unique feature being that mating surfaces of the thorium rings were plated with 0.001-in. thick chromium to prevent galling.

Tables V through VII summarize the critical cylinder specifications together with an estimate of the spherical critical mass corresponding to each cylinder measurement.

Conversions to spheres followed the same recipe as in Part I, making use of the buckling vs core composition relationships that were established with uranium reflectors.

APPENDIX: REFLECTOR SAVINGS OF VARIOUS MATERIALS ON Pu CYLINDERS

End-reflector savings on an otherwise unreflected plutonium cylinder were determined for fifteen reflector materials. Corresponding measurements

TABLE V

Critical Parameters of 2-in. Thick Thorium-Reflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
					Core-average densities (g/cc)			
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Pu	principal diluent	Ni	Estimated spherical critical mass-kg Pu
None	--	14.49	14.67	2.246	14.25	--	0.647	11.3
Depleted uranium	2:1	16.32	17.59	3.897	9.85	5.29	0.446	16.4
Steel (SS 304)	2:1	18.09	19.43	4.322	9.81	2.51	0.446	18.0
Th	2:1	18.09	19.98	4.443	9.81	3.63	0.446	18.8
Al (1100F)	2:1	18.09	20.05	4.459	9.81	0.839	0.446	19.1
Air	2:1	19.87	22.00	4.805	9.99	--	0.454	21.3
Depleted uranium	1:1	21.65	22.25	C.552	7.41	9.01	0.338	20.1
Steel	1:1	26.14	30.65	8.50	7.87	4.01	0.359	26.4
Th	1:1	26.14	33.53	9.78	7.48	5.54	0.341	27.8
Al	1:1	26.14	34.94	10.15	7.51	0.766	0.342	28.5
Air	1:1	26.14	57.72	16.56	7.60	--	0.347	37.8
Depleted uranium	1:2	26.14	<——— Not critical ———>					

TABLE VI

Critical Parameters of 4.5-in. Thick Thorium-Reflected 5.967 in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration					
					Core-average densities (g/cc)			
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Pu	principal diluent	Ni	Estimated spherical critical mass-kg Pu
None	--	12.56	13.25	2.018	14.33	--	0.656	10.0
Depleted uranium	2:1	14.38	15.28	3.420	9.75	5.94	0.446	13.5
Steel (SS 304)	2:1	16.23	16.91	3.744	9.85	2.52	0.449	15.3
Th	2:1	16.19	17.55	3.896	9.83	3.67	0.449	16.0
Al (1100F)	2:1	16.23	17.50	3.875	9.85	0.844	0.449	16.7
Air	2:1	16.23	18.24	3.991	9.97	--	0.454	17.4
Depleted uranium	1:1	18.04	18.87	5.524	7.45	9.06	0.340	17.5
Steel	1:1	22.55	23.54	6.851	7.39	3.77	0.337	21.0
Th	1:1	24.40	25.17	7.350	7.47	5.52	0.340	23.0
Al	1:1	24.37	25.15	7.353	7.46	1.28	0.340	23.8
Air	1:1	27.95	29.22	8.364	7.62	--	0.347	27.1
Depleted uranium	1:2	27.95	40.16	17.33	5.06	12.24	0.230	23.1
Steel	1:2	27.95	<——— Not critical ———>					

TABLE VII

Critical Parameters of 7.5-in. Thick Thorium-Reflected 5.967-in. Diam Pu Cylinders Diluted with Other Metals

Final measured configuration			Critical configuration						
Diluent	Nominal volume ratio of Pu to diluent	Mass Pu element -(kg)	Extrapolated critical mass -kg Pu	Critical height (including Ni)-in.	Core-average densities (g/cc): Pu	Core-average densities (g/cc): principal diluent	Core-average densities (g/cc): Ni	Estimated spherical critical mass-kg Pu
None	--	11.76	12.75	1.919	14.50	--	0.659	9.7
Depleted uranium	2:1	12.63	14.65	3.226	9.91	5.99	0.451	12.5
Steel (SS 304)	2:1	14.40	16.02	3.581	9.76	2.50	0.446	14.1
Th	2:1	16.25	16.61	3.689	9.82	3.63	0.447	15.1
Al (1100F)	2:1	16.25	16.45	3.606	9.95	0.851	0.453	15.0
Air	2:1	16.25	17.35	3.804	9.95	--	0.453	16.5
Depleted uranium	1:1	17.12	17.56	5.161	7.42	9.04	0.339	16.2
Steel	1:1	20.72	21.55	6.329	7.43	3.80	0.339	20.6
Th	1:1	22.52	23.16	6.781	7.45	5.52	0.340	21.3
Al	1:1	21.65	22.80	6.549	7.59	1.30	0.346	21.2
Air	1:1	25.21	25.75	7.341	7.65	--	0.349	24.5
Depleted uranium	1:2	27.03	30.78	13.260	5.06	12.27	0.231	21.3
Steel	1:2	27.03	← Not critical →					

TABLE VIII

End Reflector Savings of 1/2-in. Thick Disk of Various Materials on 6.0-in. O.D. Pu Cylinder

Reflector: Material	Reflector: Density (g/cm^3)	Reflector savings[a] cm Pu(95% Pu^{239}, 5% Pu^{240}) ρ = 14.27 g/cm^3	Reflector savings[a] cm U^{235} (93.5%) ρ = 18.80 g/cm^3
Mg (FS-1)	1.77	0.298 ± 0.01	0.37_8
Al (1100F)	2.72	0.353 ± 0.01	0.43_3
Ti (96.5 w/o)	4.50	0.384 ± 0.01	0.420
Th (> 99 w/o)	11.57	0.403 ± 0.005	--
Graphite (CS-312)	1.67	0.444 ± 0.01	0.59_7
Polyethylene	0.921	0.488 ± 0.01	0.69_3
Al_2O_3 (> 99 w/o)	2.62	0.490 ± 0.02	0.62_2
Steel (SS 304)	7.87	0.548 ± 0.01	--
Fe (SAE 1020)	7.78	--	0.59_4
Ni (electrolytic)	8.79	0.607 ± 0.01	0.75_4
Cu (99 - 99.5 w/o)	8.87	0.607 ± 0.01	0.75_2
Co (reagent)	8.72	0.623 ± 0.01	0.77_8
Mo (99.8 w/o)	10.53	0.635 ± 0.01	0.76_4
W (~91.3 w/o)	17.3	0.646 ± 0.01	0.81_6
U (depleted)	18.97	0.697 ± 0.005	--
U (normal)	18.8	--	0.81_4
Be (QMV)	1.84	0.701 ± 0.015	0.89_6

[a] The height to diameter ratio of the plutonium cylinders varied from 0.508 to 0.535 and for the U^{235} cylinders 1.20 to 1.69.

for these materials on U^{235} (93.5%) cylinders have been reported by Hansen *et al.* (*5*). The procedure for determining the end-reflector savings is as follows:

(1) The reciprocal-multiplication curve for the unreflected plutonium cylinder was established as previously described.

(2) Plutonium plates were removed from the top of the assembly and replaced with a ½-in. thick by 6.0-in. o.d. reflector disk, the assembly was closed and a counting rate recorded. Pu plates were added, under the reflector disk, one at a time until the safe limit of multiplication was reached to establish a reciprocal-multiplication curve for the plutonium assembly with end reflector.

(3) The difference between extrapolated critical heights of the unreflected cylinder and of the partially reflected Pu (corrected for Ni and voids), represents the reflector savings due to ½-in. thick end reflector.

Results are recorded in Table VIII, and reflector savings of ½-in. thicknesses of these materials on U^{235} (93.5%) are included for comparison.

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