REFERENCE 159

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THE NUCLEAR CRITICALITY OF INTERSECTING CYLINDERS OF AQUEOUS URANYL FLUORIDE SOLUTIONS

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UNION CARBIDE

OAK RIDGE Y-12 PLANT
OAK RIDGE, TENNESSEE

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ABSTRACT

The first experimental determinations of the criticality of aqeuous solutions of uranyl fluoride, in which the uranium was enriched to 5% in ^{235}U, in intersecting cylinders have been made. These experiments were designed to provide benchmarks for calculational methods. Two types of intersections were investigated: a 30° lateral formed by cylinders about 28 cm in diameter and a cross formed by cylinders of four diameters, between 26.7 and 28.6 cm. Solution concentrations between 907 and 745 g of uranium per liter were made critical. Comparison of KENO calculations, using two different cross-section sets, with the experiments showed generally very good agreement.

INTRODUCTION

The Oak Ridge Critical Experiments Facility reported,[1] in 1958, the first experimental data for critical and subcritical intersecting cylinders of aqueous uranyl fluoride solution in which the uranium was enriched to about 93% in ^{235}U. These data were obtained with pipes between 10.2 and 19.0 cm (4 and 7.5 in.) in diameter, reflected and not, with solution moderations near those for minimum critical mass and volume.

More than ten years later, the Rocky Flats Plant[2] of Dow Chemical Company reported additional data for intersecting pipes of aqueous uranyl nitrate solution in which the uranium was enriched to 93% in ^{235}U. These geometries were much more complex than those investigated by Oak Ridge. Correlations and calculations of the data have appeared in several publications.[3-5]

This report is concerned with what is believed to be the first series of experiments with aqueous solutions containing uranium of low enrichment in ^{235}U that were made critical in intersecting cylinders. The uranium was enriched to 5 wt% ^{235}U, and the solution concentrations ranged between about 907 and 745 g of uranium per liter of solution. Vessels of two different shapes and materials were used, both of which were reflected by an effectively infinite thickness of water except on the top of the solution columns. Some of the data were reported in Ref. 6. Subsequently, additional calculational analyses were reported by Cross *et al.*[7]

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1. J. K. Fox, L. W. Gilley, and D. Callihan, "Critical Mass Studies, Part IX. Aqueous U^{235} Solutions," ORNL-2367, Oak Ridge National Laboratory (1958).
2. Bruce B. Ernst and C. L. Schuske, "Empirical Method for Calculating Pipe Intersections Containing Fissile Solutions," RFP-1197, Dow Chemical Company (1968).
3. Deanne Dickinson, "Calculations for Pipe Intersections Containing Fissile Solution," RFP-1499, Dow Chemical Company (1970).
4. Deanne Dickinson and C. L. Schuske, *Nucl. Technol.* **10**, 179 (1971).
5. C. L. Schuske and S. J. Altschuler, *Nucl. Technol.* **18**, 305 (1973).
6. E. B. Johnson, *Trans. Am. Nucl. Soc.* **14**, 678 (1971).
7. N. F. Cross, G. E. Whitesides, and R. J. Hinton, *Trans. Am. Nucl. Soc.* **17**, 268 (1973).

EQUIPMENT AND MATERIALS

Fissile Solution

The fissile material was in the form of $U(5)O_2F_2$ in aqueous solution. The uranium had the following isotopic distribution, in percent by weight:

^{234}U: 0.03
^{235}U: 5.00
^{236}U: 0.05
^{238}U: 94.92

The average ^{235}U enrichment, based on 17 analyses, was 5.00 $\pm$ 0.05 wt %; this is the value that was used in the calculations.

Analysis showed no free fluorine and no HF; the pH was about 1.5. Impurities in the solution were determined to be, in parts per million, averaged over 20 analyses:

Cr	3	Ca	14	Fe	28
Ni	29	Cu	11	Mn	1
Al	7	Mg	6	Na	3

Trace amounts of other elements were below the level of measurement.

30° Lateral

The two cylinders forming the nominally 30° lateral were made of Type 1100 H14 aluminum and were fabricated in the Oak Ridge Gaseous Diffusion Plant (ORGDP) shops. Sheet stock 0.16-cm thick was rolled and welded to produce cylinders nominally 27.9 cm (11.0 in.) in inside diameter. The intersection of the two cylinders was formed as shown in Fig. 1. The complete unit is shown in Fig. 2. Communication between the top of the lateral and the vertical was through a 2.2-cm-i.d. tube. The top of the lateral was covered by a 0.64-cm-thick plate welded in place; the top of the vertical was open. The bottom plate of the vertical cylinder was 1.3-cm thick with a 5.1-cm-i.d. central hole that provided communication with the solution storage system. External support rings 0.635-cm thick and 2.5-cm wide were spaced at

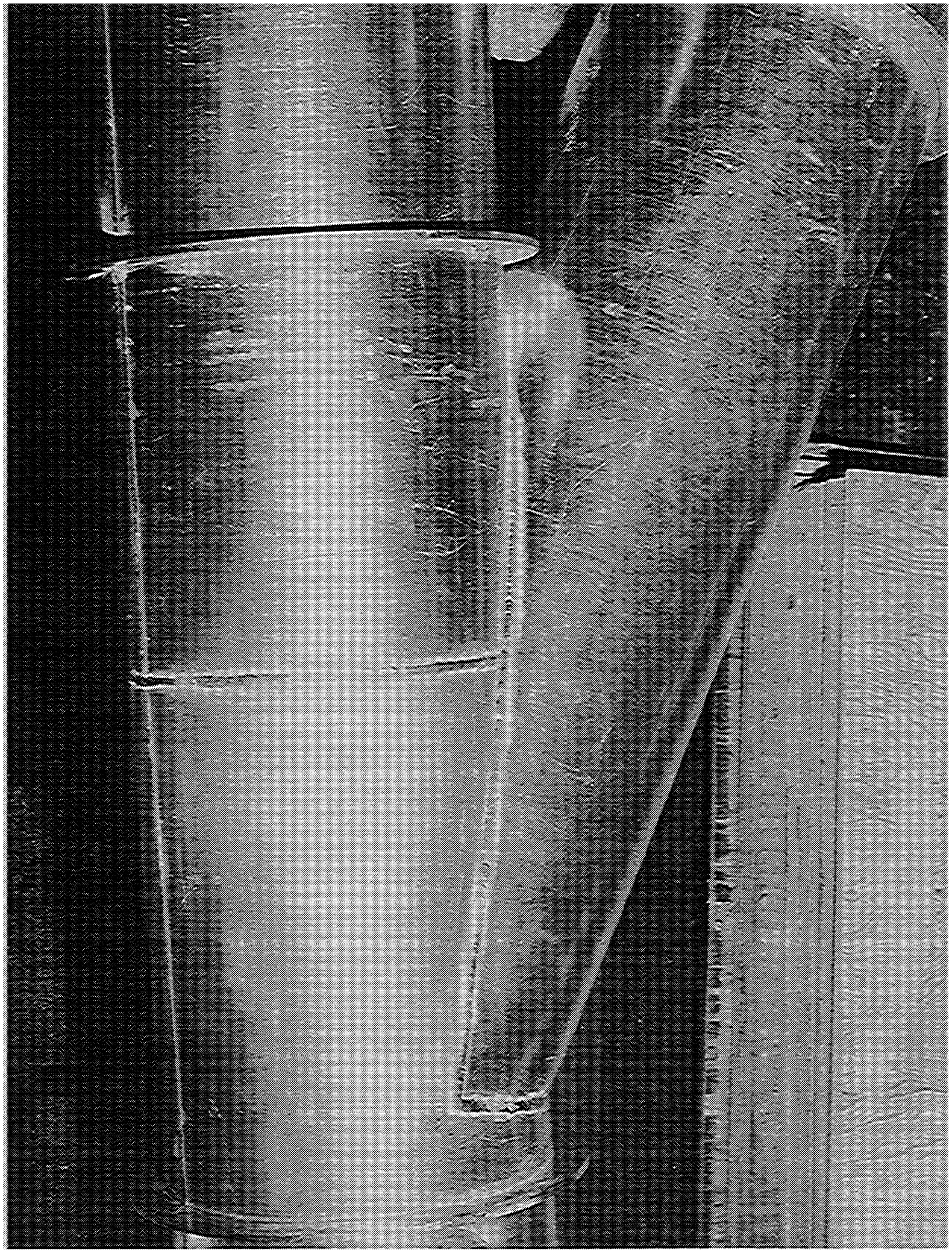

Fig. 1. The Region of Intersection of the Two Aluminum Cylinders Forming the 30° Lateral.

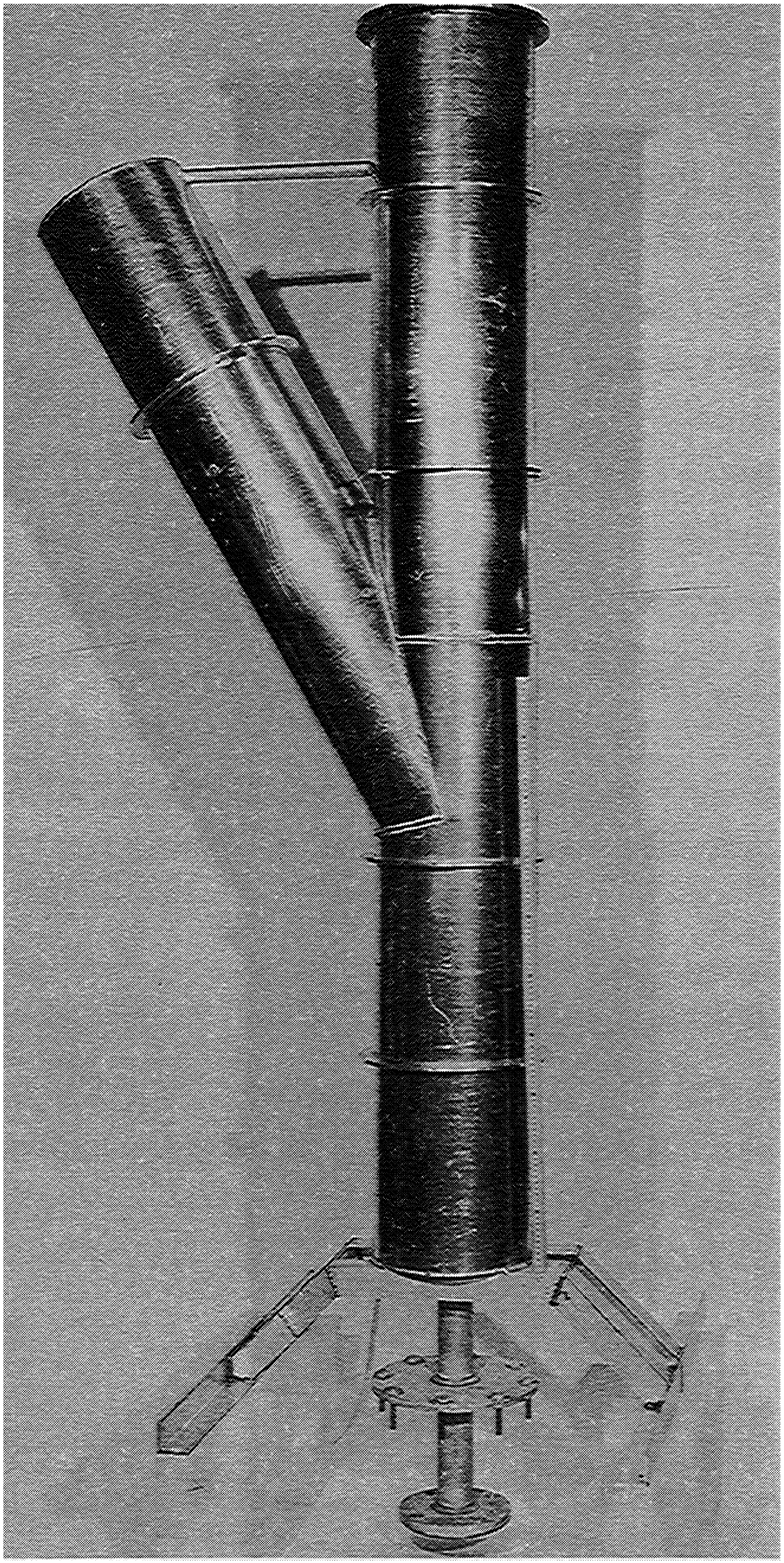

Fig. 2. The 30° Lateral Prior to Installation in the Reflector Tank.

intervals as shown in Fig. 2. The ring at the top of the vertical cylinder was 1.3-cm thick and 2.5-cm wide and was drilled appropriately to facilitate handling and positioning the unit in the reflector tank. The experimental vessel was mounted in the center of a 270-cm-diam stainless-steel-lined tank to which water could be added. As installed, the bottom of the solution in the intersecting cylinders was 22.9 cm above the floor of the reflector tank, thereby providing an effectively infinite bottom water reflector.

Calibration with water prior to installation in the system indicated that the inside diameter was not uniform. Physical measurements showed that the two cylinders were out-of-round and that the inside diameters were in the range between 27.7 and 28.4 cm for the vertical member and between 27.9 and 28.0 cm for the arm. There was a bulge at the weld between the vertical member and the arm, which was particularly unfortunate because it was near the region of maximum reactivity. This feature is evident in the photograph of Fig. 3.

Because the walls of the two cylinders were not parallel to their axes, due to distortion during welding, it was not possible to obtain a single value of the angle of intersection by direct measurement. Therefore, the effective angle between the vertical and lateral members was determined through laborious application of the volume-calibration-with-water data and the measurement of the area of intersection. These measurements and calculations indicated that the effective angle of intersection was 29.26°.

In order to prevent chemical attack of the aluminum of the vessel by the uranyl fluoride solution, the inside of the vessel was coated with Heresite P-403, which produces a ceramic-like surface after curing.[8] The inside dimensions of the unit, shown in Fig. 4, were determined with the completed unit immediately prior to use with the uranyl fluoride solutions.

8. Heresite P-403 was analyzed at the Oak Ridge Gaseous Diffusion Plant and found to contain 25.1 wt% iron and less than 0.02 ppm clorine, 5 ppm boron, and 10 ppm cadmium, the limit of detection by spectrochemical means.

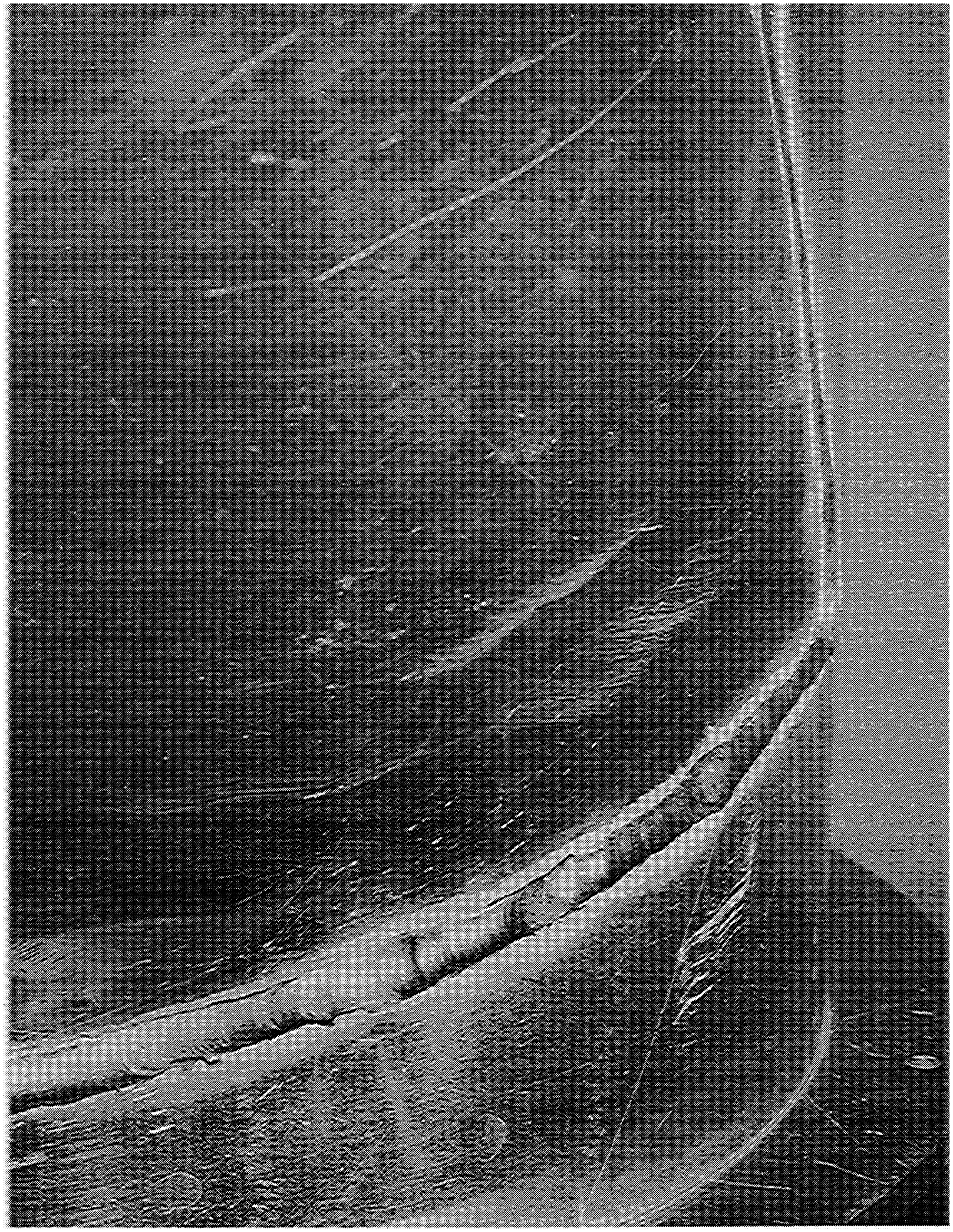

Fig. 3. Closeup of the Irregularity in the Weld at the Intersection of the Members of the 30° Lateral.

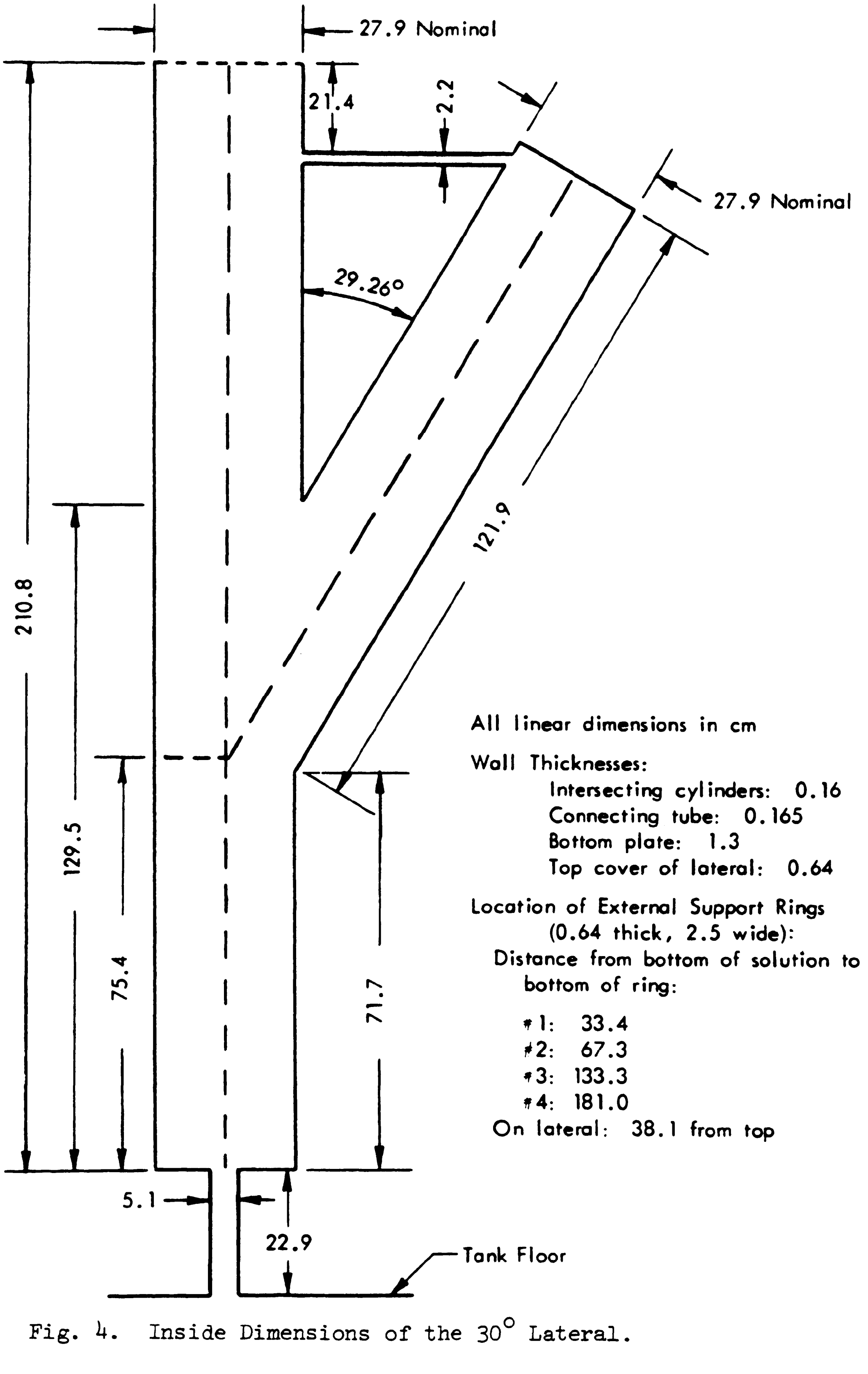

Fig. 4. Inside Dimensions of the 30° Lateral.

Cross

The vessel used to investigate the criticality of the $U(5)O_2F_2$ solution in cylinders intersecting at right angles was constructed of Plexiglas, a methacrylate, $C_5H_8O_2$, having a density of 1.18 g/cm^3. The diagram of Fig. 5 shows the relevant dimensions of the portion containing solution. The vessel was constructed from four individual cylinders with an initial machined inside diameter of 26.67 cm (10.50 in.); one end of each of these cylinders was fitted and cemented[9] so that all the centerlines intersected at a common point. The axes of the vertical members were colinear and those of the side arms were tilted 2° with the horizontal to facilitate draining. Gusset plates of Plexiglas, 2.54-cm thick, were cemented to the outside of the four sections, as shown in Fig. 5, to provide rigidity to the structure. The ends of the horizontal arms were closed with 3.17-cm-thick Plexiglas discs (not shown in Fig. 5) internally drilled and fitted so that vent lines could be attached to allow air to escape as solution entered. A similar vented blind flange covered the top of the vertical section. It was necessary that these blind flanges be removable in order to increase the inside diameter of the entire vessel. The bottom flange of the Plexiglas vessel was attached to a 1.27-cm-thick stainless steel flange into the center of which a 2-in. Schedule 40 steel pipe was welded; the nominal inside diameter of this pipe was 5.25 cm. This assembly was part of the spool piece that allowed attachment to the solution storage system and sealing to the bottom of the reflector tank.[10] A seamless stainless

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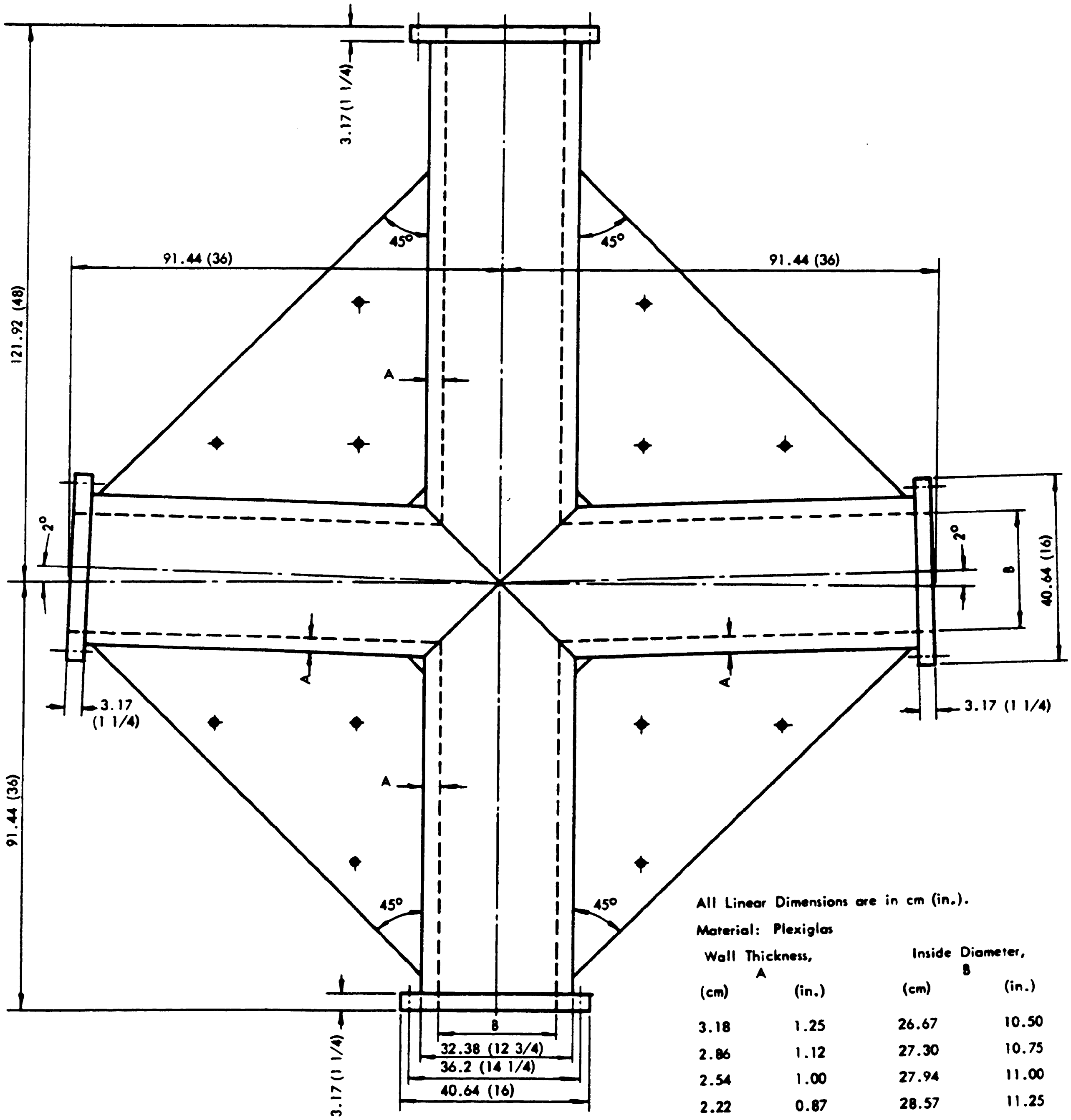
9. All permanent joints in the Plexiglas parts were sealed with an Epoxy cement. Analyses were made of both the resin and the hardener. In addition to the expected carbon, hydrogen, and oxygen, the resin contained 0.1 ppm Ca, 0.01 ppm Fe, 0.4 ppm Si, and 0.05 ppm S; the hardener contained 0.01 ppm Ca, 0.1 ppm Fe, 0.1 ppm Pt, 5 ppm Si, and 0.7 ppm S; uncertainties in these quantities are $\pm$ a factor of 3. The quantities of any other solids that might have been present were below the limit of detection.
10. The Plexiglas cross is described in Y-12 Plant Drawing C-9213-91670. The spool piece is described in an unnumbered drawing entitled "9213-CEF-5% Cross Spool Piece with Bellows," on file at the Oak Ridge Critical Experiments Facility.



All Linear Dimensions are in cm (in.).

Material: Plexiglas

Wall Thickness, A (cm)	(in.)	Inside Diameter, B (cm)	(in.)
3.18	1.25	26.67	10.50
2.86	1.12	27.30	10.75
2.54	1.00	27.94	11.00
2.22	0.87	28.57	11.25

Fig. 5. Diagram of the Plexiglas Cross.

steel bellows nominally 7.9 cm long was installed, with the top 2.54 cm below the bottom of the flange that mated with the Plexiglas flange, between sections of the 2-in. Schedule 40 pipe to provide flexibility necessary for proper alignment of the Plexiglas vessel in the reflector tank. A 2.5-in. Schedule 40 sleeve surrounded the bellows and was clamped in place with set screws after the necessary adjustments were made. The essential details of the spool piece are shown in Fig. 6. When the vessel was mounted in the reflector tank, the solution column was about 24 cm above the bottom of the reflector tank, thus providing an effectively infinite bottom water reflector with, of course, the 1.3-cm-thick steel flange intervening. The Plexiglas vessel is shown in Fig. 7.

Since corrosion attack of Plexiglas and stainless steel by aqueous uranyl fluoride solutions is negligible, there was no need to protect the contact surfaces with other inert material.

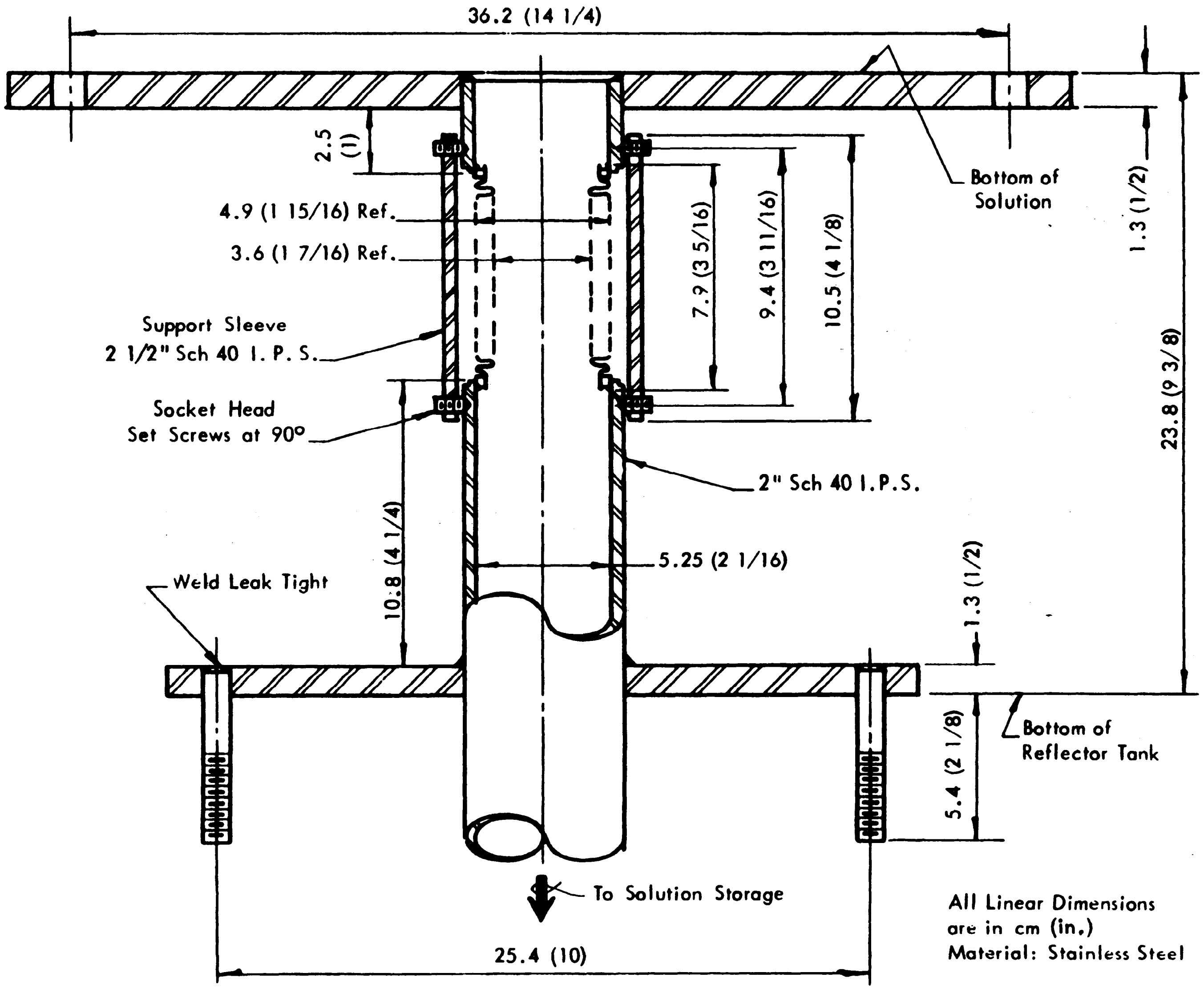


Fig. 6. Diagram of the Spool Piece that Connected the Plexiglas Cross with the Solution Storage Manifold and Sealed the Bottom of the Reflector Tank.

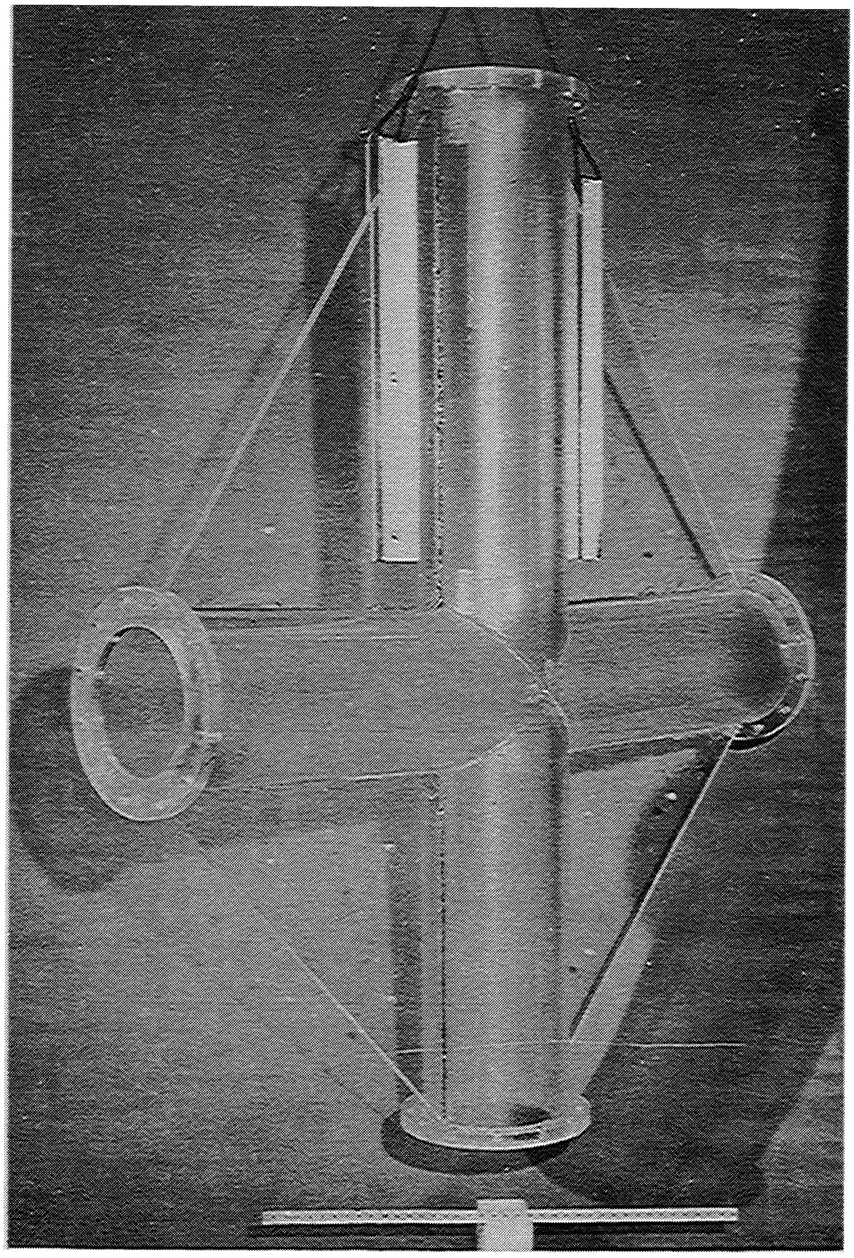


Fig. 7. The Plexiglas Cross Prior to Installation in the Reflector Tank.

EXPERIMENTS AND RESULTS

Aqueous solutions of $U(5)O_2F_2$ of six different concentrations were made critical in the 30° lateral in the presence of an effectively infinite water reflector except on the top of the vertical member. The data are presented in Table 1. Figure 8 is a plot of the height of the solution at criticality as a function of uranium concentration. The solution critical heights ranged between just below the top of the intersection of the lateral with the vertical to about 40 cm above this intersection. The height of the reflector water with respect to the height of the solution made little difference in the critical solution height.

Solutions of three different concentrations were made critical in the Plexiglas cross when the inside diameter of all members was 26.67 cm (10.5 in.). In this case also the height of the reflector water, i.e., whether its level was the same as or above the level of solution, had little influence on the critical height of the solution. Even at the highest solution concentration, the critical height was above the top of the intersection of the horizontal arms with the vertical. At the conclusion of these measurements, the inside diameter of all members was increased in three steps of 0.64 cm (0.25 in.) each to a final diameter of 28.57 cm (11.25 in.). The data for all four diameters are presented in Table 2. As the diameters were increased (and the thickness of the Plexiglas wall decreased), the influence of the height of the reflector water became greater, particularly at the higher solution concentrations. In the 27.94-cm-diam geometry, the solution was critical 0.6 cm above the top of the solution intersection, thus forming essentially a "T"; the solution concentration in this case was 904.6 g of uranium per liter. When the diameter was increased to 28.57 cm (11.25 in.), the surface of the critical solution was about a centimeter below the top of the horizontal arms, thereby producing an essentially horizontal cylinder with a flat upper surface. Figure 9 is a plot of the critical data for each of the four diameters investigated; in every case represented here, the reflector water was essentially at the bottom of the top ring of the vertical arm. The

Table 1. Summary of Critical Conditions of the 30° Lateral of $U(5)O_2F_2$ Aqueous Solution.

Height of Reflector Water[a] (cm)	Solution: Critical Height (cm)	Solution: Δ Height for Positive Period (cm)	Reactivity (cents)	Sensitivity ($\Delta\rho/\Delta h$, cents/cm)	Temperature (°C): Solution[b]	Temperature (°C): Water	Calculated k_{eff}: Hansen-Roach 16-Group Cross Sections	Calculated k_{eff}: XSDRN 123-Group Cross Sections
Solution Concentration: 907.0 g of U/liter; Density: 2.0289 g/cm³								
210.1	128.2	0.40	9.3	23.2	26.1	26.0	0.9850 ± 0.0061	0.9866 ± 0.0049
209.8	128.4[c]	0.40	11.4	28.5	26.0	26.2		
209.8	128.3[c]	0.60	15.1	25.2	26.0	26.2		
144.9	128.55	0.50	12.4	24.8	26.1	26.0		
144.9	128.45	0.35	9.0	25.7	25.5	25.9		
129.2	129.1	0.55	10.6	19.3	25.5	25.9		
Solution Concentration: 885.8 g of U/liter; Density: 2.0048 g/cm³								
210.2	131.35	0.50	10.6	21.2	26.4	--	0.9959 ± 0.0061	0.9958 ± 0.0042
147.2	131.6	--	--	--		--		
132.0	132.2	--	--	--		--		
Solution Concentration: 876.5 g of U/liter; Density: 1.9952 g/cm³								
210.2	132.1	0.65	12.2	18.8	25.6	26.0	0.9974 ± 0.0057	1.0116 ± 0.0058
147.5	132.4	0.60	10.1	16.8		--		
132.8	132.2	1.50	8.3	5.5		26.0		
Solution Concentration: 824.8 g of U/liter; Density: 1.9367 g/cm³								
209.9	141.65	1.95	14.7	7.5	25.6	26.0	1.0003 ± 0.0060	0.9916 ± 0.0053
157.5	141.9	1.60	12.4	7.7		26.0		
143.2	142.75	2.30	13.6	5.9		26.0		

Table 1 (Cont'd).

Height of Reflector Water[a] (cm)	Solution: Critical Height (cm)	Solution: Δ Height for Positive Period (cm)	Solution: Reactivity (cents)	Solution: Sensitivity ($\Delta\rho/\Delta h$, cents/cm)	Temperature (°C): Solution[b]	Temperature (°C): Water	Calculated k_{eff}: Hansen-Roach 16-Group Cross Sections	Calculated k_{eff}: XSDRN 123-Group Cross Sections
Solution Concentration: 801.1 g of U/liter; Density: 1.9108 g/cm³								
210.1	155.25	6.10	10.2	1.7	25.8	26.0	1.0102 ± 0.0056	0.9964 ± 0.0051
171.9	155.35	3.65	6.9	1.9		--		
155.9	155.85	7.35	7.9	1.1		--		
Solution Concentration: 796.2 g of U/liter; Density: 1.9059 g/cm³								
210.1	169.10	33.6	4.9	0.15	25.9	--	1.0043 ± 0.0045	0.9941 ± 0.0055
185.5	168.00	9.2	4.3	0.45		--		
168.1	168.20	32.8	3.8	0.12		--		
Solution Concentration: 774.6 g of U/liter; Density: 1.8823 g/cm³								
	(204.60)	Subcritical			26.3	26.2		

a. The water height measured from the same reference as was the solution height; i.e., from the top of the inside surface of the bottom plate of the lateral.

b. The solution temperatures were determined either before or after establishing the solution critical height or at the beginning and end of a short series. A single entry in this column is the average solution temperature for that concentration. The reported critical heights were determined with no thermocouples in the solution.

c. These two runs were made to establish the worth of three additional aluminum rings, each 0.25-in.-thick, placed on the lower vertical section of the lateral.

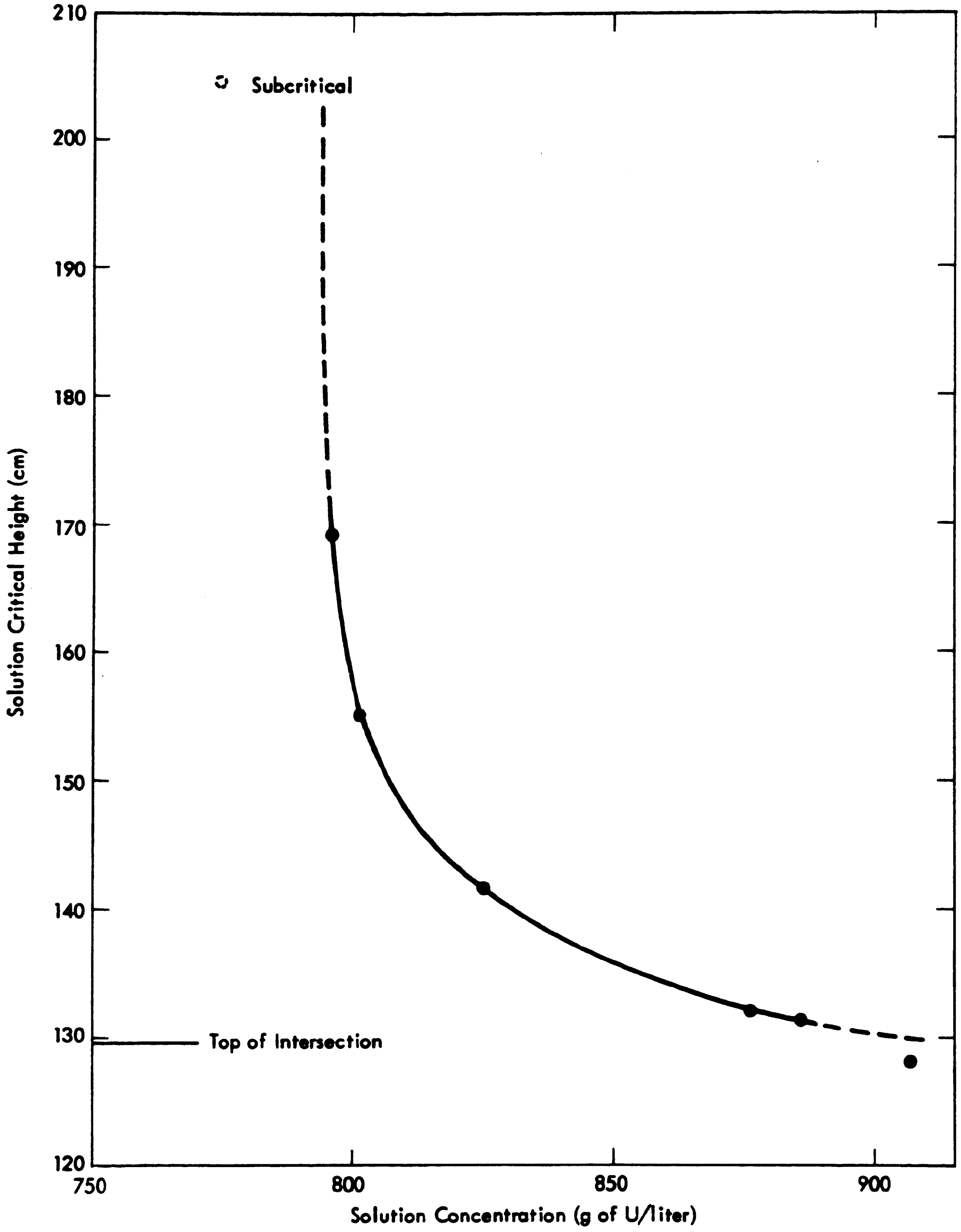


Fig. 8. Effect of Solution Concentration on Criticality in the 30° Lateral.

Table 2. Summary of $U(5)O_2F_2$ Aqueous Solution Data in Plexiglas Cross.

Nominal Inside Diameter (in.)	Nominal Inside Diameter (cm)	Solution[a] Concentration (g of U/liter)	Solution[a] Density (g/cm³)	Solution[a] Critical Height (cm)	Solution[a] Height Above Center of Intersection (cm)	Height of Water (cm)	Temperature (°C) Solution	Temperature (°C) Water	Calculated k_{eff} Hansen-Roach 16-Group Cross Sections	Calculated k_{eff} XSDRN 123-Group Cross Sections
10.50[b]	26.67[b]	904.85	2.020	115.45	24.0	210.5[c]	24.2	22.7		
				115.6	24.2	210.5[c]	25.7	22.7	0.9924 ± 0.0065[d]	1.0047 ± 0.0063[d]
				115.9	24.5	115.9[e]	24.6	22.7		
				116.0	24.6	116.0[e]	25.7	22.7		
		896.1	2.015	117.2	25.8	117.3	25.0	23.1	1.0027 ± 0.0059	1.0061 ± 0.0056
		856.4	1.970	133.9	42.5	210.5[c]	24.9	23.1	0.9929 ± 0.0069	0.9904 ± 0.0049
				134.1	42.7	134.1[e]	24.9	23.1		
10.75[f]	27.30[f]	906.4	2.026	109.6	18.2	210.5[c]	24.4	23.1	0.9864 ± 0.0061	0.9845 ± 0.0048
				110.6	19.2	110.6[e]	24.4	23.1		
		857.35	1.971	115.55	24.1	210.5[c]	24.7	22.8	1.0005 ± 0.0058	0.9957 ± 0.0052
				116.05	24.6	116.05[e]	24.7	22.8		
11.00[g]	27.94[g]	904.6	2.024	106.3	14.9	210.5[c]	24.8	23.1	1.0111 ± 0.0057	0.9840 ± 0.0058
				107.9	16.5	107.9[e]	24.8	23.1		1.0168 ± 0.0056
		859.5	1.971	110.0	18.6	210.5[c]	24.3	23.0	0.9931 ± 0.0053	1.0009 ± 0.0057
				110.95	19.5	110.95[e]	24.6	23.0		
		811.2	1.920	117.0	25.6	210.5[c]	24.7	23.5	1.0047 ± 0.0061	1.0086 ± 0.0053
				117.5	26.1	117.5[e]	24.7	23.5		

Table 2 (Cont'd).

Nominal Inside Diameter (in.)	Nominal Inside Diameter (cm)	Solution[a] Concentration (g of U/liter)	Solution[a] Density (g/cm^3)	Solution[a] Critical Height (cm)	Solution[a] Height Above Center of Intersection (cm)	Height of Water (cm)	Temperature (°C) Solution	Temperature (°C) Water	Calculated k_{eff} Hansen-Roach 16-Group Cross Sections	Calculated k_{eff} XSDRN 123-Group Cross Sections
11.25[h]	28.57[h]	905.3	2.023	104.45	13.0	210.5[c]	25.1	25.0	0.9849 ± 0.0062	0.9973 ± 0.0062
				105.7	14.3	105.7[e]	25.1	25.0	0.9863 ± 0.0056	0.9978 ± 0.0060
		854.9	1.967	107.8	16.4	210.5[c]	25.7	26.3	1.0031 ± 0.005	1.0035 ± 0.006
				109.0	17.6	109.0[e]	25.7	26.3		
		812.8	1.921	111.0	19.6	210.5[c]	26.5	27.9	0.9946 ± 0.0052	1.0047 ± 0.0063
				111.9	20.5	111.9[e]	26.5	27.9		
		786.4	1.892	115.4	24.0	210.5[c]	27.3	27.6	1.0043 ± 0.0057	1.0039 ± 0.0054
				115.95	24.5	115.95[e]	27.3	27.6		
		764.2	1.866	122.65	31.2	210.5[c]	25.7	27.0	1.0064 ± 0.0055	1.0057 ± 0.0056
				123.15	31.7	123.15[e]	25.7	27.0		
		744.9	1.844	150.05	58.6	210.5[c]	25.6	26.2	0.9873 ± 0.0053	0.9937 ± 0.0054
				150.05	58.6	150.05[e]	25.6	26.2		

a. The values quoted for both solution concentration and solution density are the average of two determinations by different laboratories.

b. On the basis of the calibration data, the average inside radius of the lower vertical cylinder is 13.327 cm and of the top vertical cylinder 13.341 cm, corresponding to an average diameter of 10.494 in. and 10.505 in., respectively.

c. The water height is measured from the same reference as the solution; i.e., at the bottom of the lower vertical member, 91.4 cm below the intersection centerline. At a height of 210.5 cm the water surface was at the bottom of the top flange.

d. The uncertainties are one standard deviation of the eigenvalue resulting from the statistical nature of Monte Carlo codes.

e. In these cases, criticality was established with the solution and water at the same height as determined by a carpenter's level across the indicated heights in the two manometers.

f. On the basis of the calibration data, the average inside radius of the lower vertical cylinder was 13.63 cm and of the top vertical cylinder 13.667 cm, corresponding to an average inside diameter of 10.735 in. and 10.761 in., respectively.

g. On the basis of the calibration data, the average inside radius of the lower vertical cylinder was 13.957 cm and of the top vertical cylinder 13.963 cm, corresponding to an average inside diameter of 10.990 in. and 10.994 in., respectively.

h. On the basis of the calibration data, the average inside radius of the lower vertical cylinder was 14.273 cm and of the top vertical cylinder 14.286 cm, corresponding to an average inside diameter of 11.239 in. and 11.249 in., respectively.

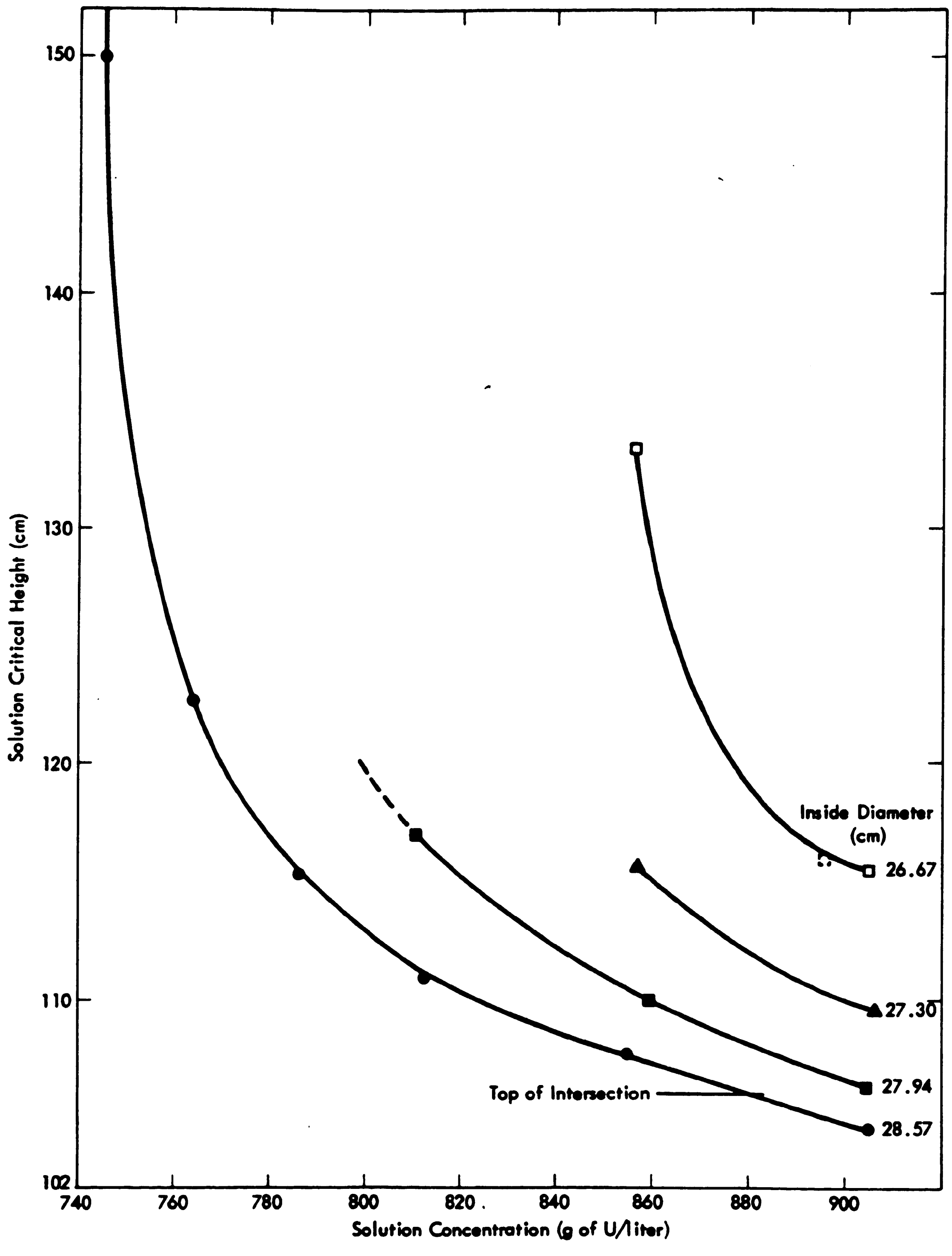


Fig. 9. Criticality of $U(5)O_2F_2$ Solution in a Cross of Different Diameters as a Function of Solution Concentration.

information is replotted in Fig. 10 in order to depict the geometry necessary for the criticality of solutions of several concentrations.

At the conclusion of the experiments with full water reflector, a test intended to simulate piping passing through a concrete wall, then branching, was performed. The water was brought to the level of the bottom of the intersection of the horizontal arms with the central column (77.2 cm from the bottom of the solution column) and solution was added to a height 84.5 cm above the center of intersection (175.9 cm from the bottom of the solution column). The inside diameter of the cross was 28.57 cm and the solution concentration was 905.3 g of uranium per liter. The solution was subcritical under these conditions and also when the water level was raised 2.3 cm to the bottom of the ends of the horizontal arms. Increasing the water height 7.5 cm more resulted in criticality. This brief series of experiments demonstrated that the solution would be subcritical in a cross of these dimensions in the absence of a water reflector.

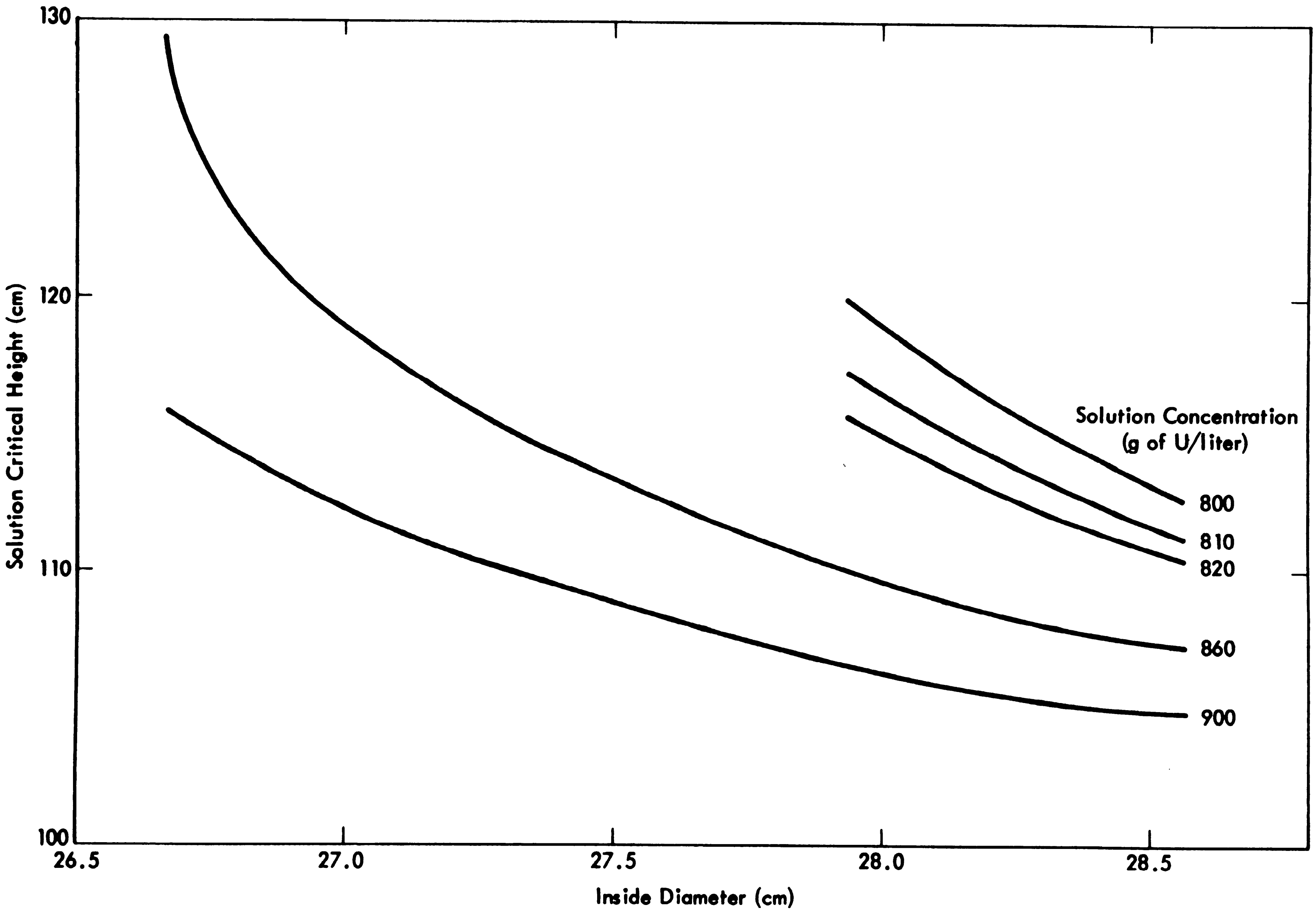


Fig. 10. Criticality of $U(5)O_2F_2$ Solution as a Function of Diameter in Cylinders Intersecting to Form a Cross.

CALCULATIONS

Computer calculations were employed to an unusual extent during the course of these experiments. In addition to the customary calculation of experimentally critical systems, they were also used to evaluate the effect on the neutron multiplication factor of the irregularities in the 30° lateral. As was mentioned above, the effective angle of intersection was determined, by a combination of physical measurements and calculations, to be 29.26°. It was estimated that, had the angle of intersection actually been 30°, the value of k_{eff} would have been about 0.6% less. In the same vessel there were nonuniformities in the actual radius. The ANISN code[10] was used to evaluate the possible effects of a change in radius by running a series of single infinitely long cylinders of fuel reflected by 6 in. of water. The results indicate that changes in radius of 0.13 cm (0.05 in.) and 0.32 cm (0.125 in.) produce a Δk of 0.0038 and 0.0093, respectively; the k_{eff} of the reference case was 0.9923 for a radius of 14.74 cm. The fuel utilized in these calculations contained 906.4 g of U/liter. It was concluded that the Δk resulting from nonuniformities in the pipe are well within the deviation associated with the calculations. The calculations reported in Table 1 assumed an angle of intersection of 29.26° and uniform radii.

The design of the Plexiglas cross was established on the basis of extensive computer calculations. In addition to the specification of inside diameters that would permit flexibility in data acquisition, other details that were investigated calculationally prior to completion of the design were the length of the intersecting arms, the acceptability of the Plexiglas gussets that were used to provide rigidity, and the effects of various heights of the water reflector with respect to the expected critical solution heights.

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11. Ward W. Engle, "A Users Manual for ANISN, A One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," K-1693, UCC-ND, Oak Ridge Gaseous Diffusion Plant (1967).

The calculated values of k_{eff} shown in Tables 1 and 2 were obtained with the KENO[12] Monte Carlo code. It is evident that the calculations demonstrate reasonably well the applicability of the Hansen-Roach[13] 16-group cross sections to homogeneous critical systems of low ^{235}U enrichment. If there is a trend in these data, it appears that these cross sections give slightly nonconservative results at the higher solution concentrations and are more conservative at lower concentrations.

The calculations were repeated using 123-group cross sections. These multigroup data were obtained from the ENDF Version II cross sections reduced in energy using the SUPERTOG code with 1/E weighting.[14] The XSDRN code[15] was used to produce the proper resonance correction for each solution concentration. Of the 123 energy groups, 30 were in the thermal range, i.e., from 1.8 eV downward. The results of calculations with this cross-section set were quite comparable with those using the 16-group set, except that there seemed to be no trend with concentration.

Some measure of how well the calculations can reproduce the critical experiments in this range of concentrations can be obtained by averaging all the calculated results for each cross-section set. The average k_{eff} for the 16-group set was 0.9973 and for the 123-group set 0.9990. These averages are in excellent agreement with the experimentally critical systems but there are a few individual results that are more than three standard deviations away from the average. Figures 11 and 12 show graphically the calculated values of k_{eff} that were obtained using the two cross-section sets.

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12. G. E. Whitesides and N. F. Cross, "KENO-A Multigroup Monte Carlo Criticality Program," CTC-5, Oak Ridge Computing Technology Center (1969).
13. Gordon E. Hansen and William H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1961).
14. C. W. Craven, Jr., et al., "SUPERTOG: A Program to Generate Fine Group Constants and P_n Scattering Matrices from ENDF/B," ORNL-TM-2679, Oak Ridge National Laboratory (1969).
15. N. M. Greene and C. W. Craven, Jr., "XSDRN: A Discrete Ordinates Spectral Averaging Code," ORNL-TM-2500, Oak Ridge National Laboratory (1969).

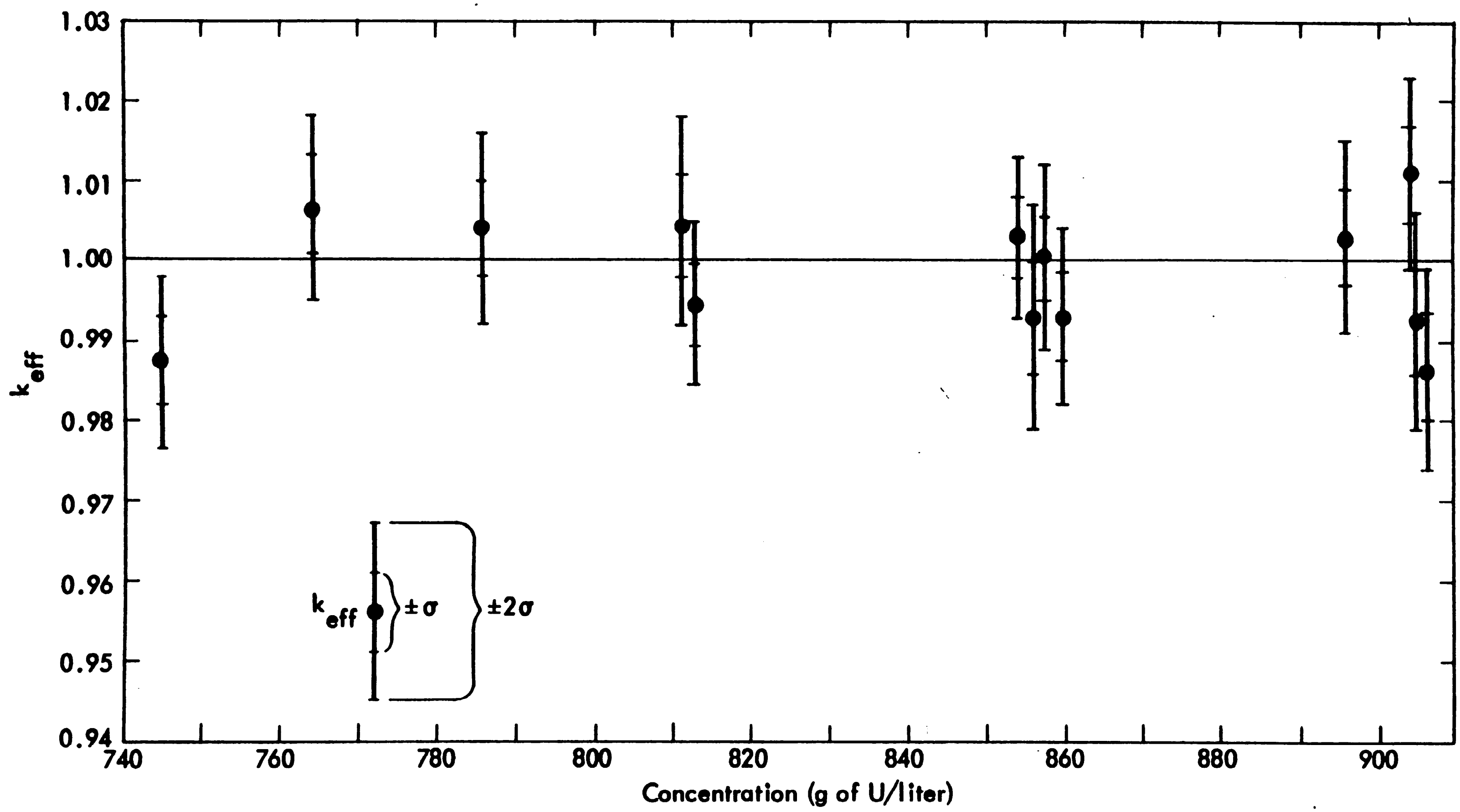

Fig. 11. Calculated k_{eff} Using Hansen-Roach Cross Sections as a Function of Concentration.

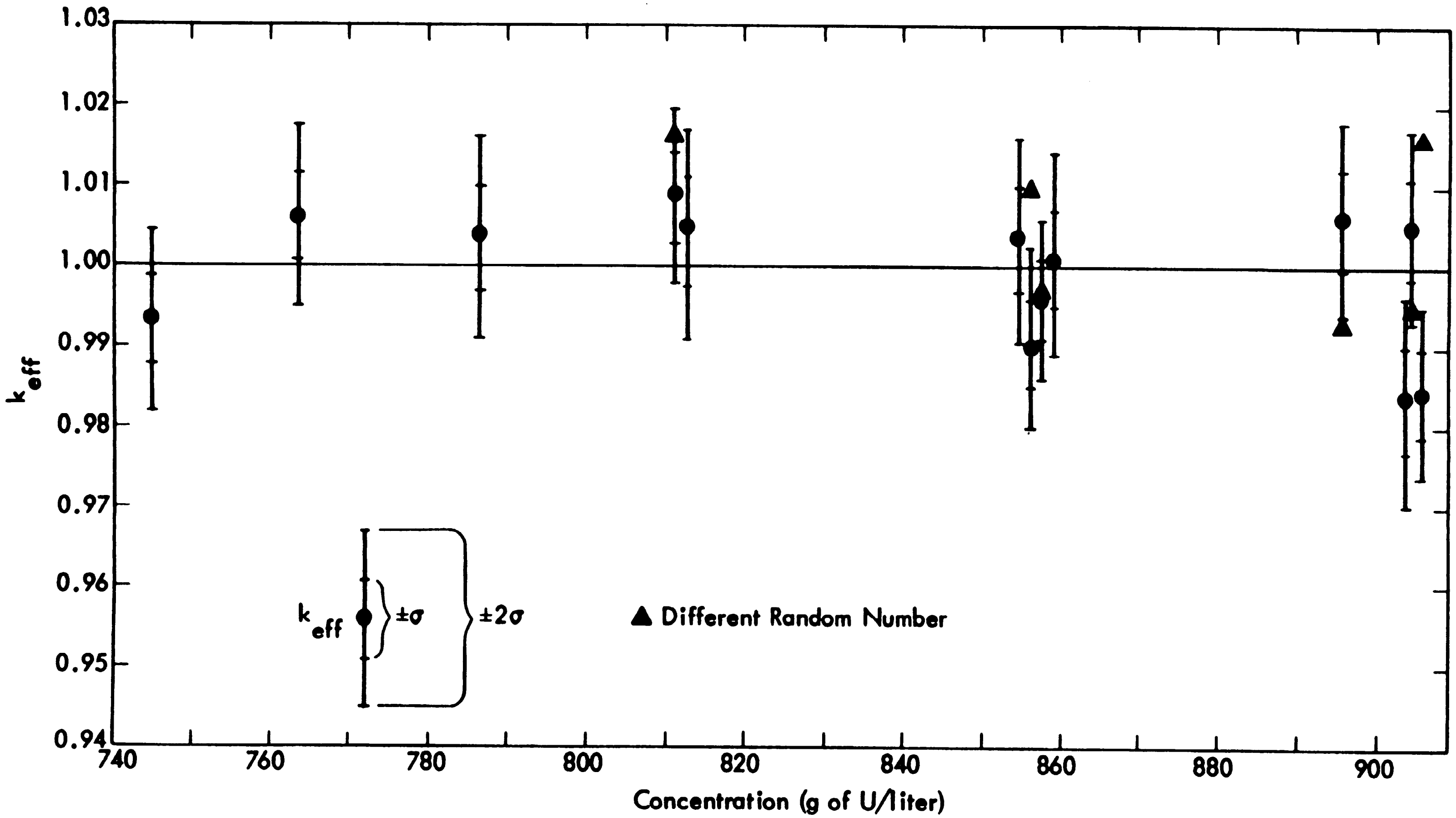

Fig. 12. Calculated k_{eff} Using 123-Group Cross Sections as a Function of Concentration.

If the cross-section set that is utilized in a Monte Carlo calculation has been validated for that type of system and there are no source convergence problems, the calculated value of k_{eff} has a 67% chance of being within one standard deviation of the true value and a 95% chance of being within two standard deviations of the true value. A single calculation may fall outside a particular confidence interval and that fact not be recognized in the absence of more calculations.

The problem of source convergence[16] is related to how well the initial source distribution used in the problem approximates the actual source distribution and how rapidly the starting distribution converges to the actual source distribution. Source convergence is affected by the location at which neutrons are introduced in the problem, the geometric configuration of the problem, and how tightly the system is coupled. The pipe intersection problems are quite sensitive to the location at which a neutron is introduced; in the present cases, it was found that the neutrons should be introduced into the region of greatest importance, i.e., in the region of the intersections.

As a result of this combined experimental-calculational effort, it is felt that it should be possible to calculate the criticality of moderately complex homogeneous systems at this uranium enrichment provided due care is exercised in setting up the problems.

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16. G. E. Whitesides, Trans. Am. Nucl. Soc. **14**, 680 (1971).

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