CRITICALITY CONTROL OF FISSILE MATERIALS

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A SIMPLE, PRACTICAL METHOD FOR CALCULATING INTERACTION*

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Abstract

A SIMPLE, PRACTICAL METHOD FOR CALCULATING INTERACTION. A review is given of a simple practical method for computing the interaction within an assemblage of fissile units, each of which would be subcritical by itself. The assemblage is not restricted to regular arrays of identical units, but may consist of dissimilar, irregularly spaced units, and may be surrounded by or in proximity to a reflector. Several approximations are necessarily made, which have been shown to be reasonable. Absolute accuracy is not essential, however, because of the large body of data now in existence that provides normalization for the method. Several comparisons of the method with experiment are presented. The problem is divided into the calculation of the nuclear properties of individual units and the purely geometrical calculation of the interchange of neutrons between units. The required nuclear properties can be obtained quite simply from critical experiments performed with units composed of the same materials or from calculations normalized to experiment. The calculation of the interchange of neutrons between units provides a boundary condition at the surface of a unit that can be expressed as the ratio of the neutron current entering the unit to the current leaving the unit, a ratio that depends on relative dimensions and spacings of the units and on the albedo of any surrounding reflector. Some of the integrals involved in calculating the interchange can be evaluated analytically, and several results of this type are presented. Others require numerical evaluation. For the latter integrals and for large arrays computing machinery is necessary, but calculations can be made once and for all in the range of interest and curves can be plotted which obviate the need for a computer in future calculations. A number of tables are presented from which such curves can be constructed.

INTRODUCTION

Fissile material encountered in operations performed outside reactors usually exist in the form of discrete units. In some cases, for example in storage, units may be identical and may be regularly spaced. In other cases, for example in a process line, units may have different sizes, shapes and compositions, and may be irregularly spaced. In all cases reflectors may be actually or potentially present. A survey of normal and credible abnormal conditions, such as is required to establish the nuclear safety of an operation, therefore generally requires reliable estimates of the interaction within groups of units and reflectors.

The general calculation of interaction is a complicated problem. Any exact solution is impracticable without the use of a modern high-speed computer, and even then is time-consuming and hence expensive. A few reasonable, simplifying approximations can be made, however, which greatly reduce the complexity of the problem. Absolute accuracy is not of prime importance

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since, regardless of the complexity of the method, its reliability should be established by comparison with experiment, and discrepancies between experiment and calculation can be factored into the calculations as a normalizing parameter. The present paper reviews a simple, practical method that has been in use for several years [1]. Tables are presented, and from these curves may be plotted, allowing simple hand calculations to be made to estimate the interaction in many cases of practical interest. Numerous comparisons are made with experiment to indicate the accuracy that can be expected.

DESCRIPTION OF METHOD

The problem is divided into:

(a) The purely geometrical calculation of the probabilities of neutron transmission from each unit in a group to each of the other units (including reflectors);

(b) The calculation of the "criticality factor" of a unit from a boundary condition expressed in terms of the ratio of the neutrons entering the unit to the neutrons leaving the unit, as determined by the above probabilities.

This division of the problem can always be made without introducing any approximation, but it results in a simplification only if approximations are made regarding (1) distribution of neutron current over transmitting and receiving surfaces, and (2) angular distribution of neutrons transmitted from an element of surface.

Two approximations made here for performing the geometrical calculation of probability of transmission from one unit to another are:

(i) Transmitted current is treated as though it were uniform over the entire surface of a sphere or cylinder and over either of the two principal surfaces of a slab.

(ii) The fraction of neutrons transmitted from an element of surface per unit solid angle is assumed to be $\cos\theta/\pi$, where θ is the angle between the direction of neutron travel and the normal to the element of surface.

The reasonableness of these approximations has been discussed elsewhere [1]. For a slab and along the length of a cylinder the assumption of a uniform current tends to compensate for ignoring the actual skewness of the angular distribution.

Criticality calculations are made in one dimension only, i.e. radial for a sphere or cylinder, or perpendicular to the two principal surfaces for a slab. If a slab is so oriented that neutrons are transmitted to an end as well as to its principal surfaces, the neutrons are treated as though they all enter the principal surfaces; neutrons transmitted from the end to other units are treated as though they were transmitted from the principal surfaces. Separability is assumed in slabs and cylinders; transmitted and received currents are treated as though their distributions were the same as the distribution of current transmitted from an isolated unit, i.e. uniform for a sphere, proportional to ($\cos B_z z$) along the length of a cylinder, and

proportional to $(\cos B_z z)(\cos B_y y)$ across the surface of a slab, where

$$B_z^2 = \frac{\pi^2}{(H + 2S_H)^2}, \quad B_y^2 = \frac{\pi^2}{(L + 2S_L)^2},$$

and S_H and S_L are appropriate extrapolation distances.

Some care must be exercised in applying these approximations. For closely spaced units, the current is clearly greater on portions of surfaces facing each other than elsewhere. Conservative approximations (such as subdividing units) could be introduced to allow for this effect. There are, however, sufficient experimental data now available to permit estimates to be made of its magnitude.

GEOMETRICAL CALCULATION

On the basis of these approximations, the probability, ρ_{jk}, that neutrons emitted from a surface k reach a surface j, is

$$\rho_{jk} = \int_{A_j} \int_{A_k} \frac{\cos\theta_j \cos\theta_k \, dA_j \, dA_k}{\pi R^2 A_k} \tag{1}$$

where A_k is the entire area of transmitting surface and $\frac{dA_j \cos\theta_j}{R^2}$ is the element of solid angle subtended by an element of receiving surface dA_j, at dA_k. Integration is carried out over the entire transmitting and receiving surfaces. This integral has been evaluated for a number of cases either analytically or numerically by quadrature.

Analytical results [1]

For two infinitely-long, parallel, rectangular surfaces having height, h, and separation, s,

$$\rho = \sqrt{1 + (s/h)^2} - s/h \tag{2}$$

For two parallel coaxial discs having transmitter and receiver radii a and b, and separation s,

$$\rho = \tfrac{1}{2}\{1 + (b/a)^2 + (s/a)^2 - \sqrt{[1 - (b/a)^2 - (s/a)^2]^2 + 4(s/a)^2}\} \tag{3}$$

For two perpendicular rectangular surfaces having transmitter height 2a, and length 2h; having receiver width 2d, and length 2g; having a separation b between the lower edge of the transmitter and the plane of the receiver; having a separation f between the centres of the transmitter and receiver measured in the common direction of 2h and 2g; and having a separation e

between the projection of the transmitter on the plane of the receiver and the centre of the receiver

$$\rho = \frac{1}{8\pi ah} \sum \left\{ G(z,z')\, H(x,y') \tan^{-1} \frac{G(z,z')}{H(x,y')} + \frac{G(z,z')^2}{4} \log \left[\frac{G(z,z')^2 + H(x,y')^2}{G(z,z')^2} \right] - \frac{H(x,y')^2}{4} \log \left[\frac{G(z,z')^2 + H(x,y')^2}{H(x,y')^2} \right] \right\} \quad (4)$$

where

$$G(z,z') = z + f - z'$$

$$H(x,y') = (x+e)^2 + (y'+a+b)^2$$

Summation (with proper regard to sign) is over the 16 terms resulting from evaluating this quadruple integral at the limits

-a and a for y'
-h and h for z'
-e (or -d if e >d) and d for x
-g and g for z

The probability of neutrons being transmitted from a rectangular surface having height 2a, and length 2h, to an identical parallel rectangular surface separated from it by a distance 2d, is clearly obtained by subtracting from unity twice the sum of the probabilities of reaching perpendicular rectangular surfaces, one having a heigth 2a, and a width 2d, and the other a width 2d, and a length 2h. For each case Eq. (4) reduces to only three terms. Table I gives probabilities for parallel rectangular surfaces as a function of the ratio of the shorter to the longer side and of the ratio of the separation to the shorter side.

Evaluation of Eq. (1) by subtracting from unity the probabilities of neutrons reaching surfaces other than the receiving surface of interest is frequently useful. For example, the probability of transmission from the inner surface of a cylinder to itself is obtained by subtracting from unity twice the probability of reaching the disc capping either end. From the reciprocity inherent in Eq. (1), this latter probability is obtained by subtracting from unity the probability of transmission from one disc to the other and multiplying the result by the ratio of the areas of the disc and cylinder. The resulting self-transmission probability for the inner surface of a cylinder of height, h, and diameter, d, is

$$\rho = 1 + \frac{h}{d} - \sqrt{1 + \frac{h^2}{d^2}} \quad (5)$$

TABLE I

Probability of Transmission, ρ, Between Two Identical Parallel Rectangular Surfaces of Height, H, and Length, L, at Separation, S

	ρ at H/L of				
S/H	1.0	0.8	0.6	0.4	0.2
0.20	0.6902	0.7151	0.7407	0.7667	0.7932
0.40	0.4892	0.5230	0.5591	0.5972	0.6367
0.60	0.3545	0.3896	0.4289	0.4720	0.5182
0.80	0.2630	0.2958	0.3344	0.3788	0.4282
1.00	0.1998	0.2291	0.2651	0.3086	0.3592
1.25	0.1464	0.1709	0.2027	0.2436	0.2939
1.60	0.0998	0.1187	0.1447	0.1806	0.2288
2.00	0.0686	0.0827	0.1031	0.1333	0.1779
2.50	0.0461	0.0562	0.0714	0.0955	0.1351
4.00	0.0191	0.0236	0.0308	0.0436	0.0700
6.25	0.0080	0.0100	0.0132	0.0192	0.0341
10.00	0.0032	0.0039	0.0052	0.0078	0.0147

A similar approach can be used when a reflector surrounds an array. The probability of reaching the reflector is unity minus the sum of the probabilities of reaching other units; the probability of transmission from the reflector to the unit is the product of this probability and the ratio of the area of the unit to that of the reflector.

Quadrature results

For a pair of identical spheres Eq. (1) has been reduced [2] to a double integral which has been evaluated by Gaussian quadrature with the results given in Table II. For a pair of identical cylinders having height, h; diameter, d; and axis-to-axis separation, s; a double integral has also been obtained,

$$\rho = \frac{1}{2\pi^2}\int_{-1}^{1}\int_{-1}^{1} \frac{\tan^{-1}\left[\dfrac{2h/d}{\sqrt{4s^2/d^2 - (u+v)^2}\,\sqrt{1-u^2}\,\sqrt{1-v^2}}\right]}{\sqrt{4s^2/d^2 - (u+v)^2}}\,du\,dv \qquad (6)$$

which has been evaluated by Gaussian quadrature with the results given in Table III.

TABLE II

Probability of Transmission, ρ, Between Two Identical Spheres

Diameter/Pitch	ρ
1.0	0.0782
0.8	0.0461
0.6	0.0247
0.4	0.0106
0.2	0.0026

TABLE III

Probability of Transmission, ρ, Between Two Identical Cylinders

	ρ at Diameter/Height Ratio of:		
Diameter/Pitch	0	0.5	1.0
1.0	0.1817	0.1753	0.1692
0.8	0.1360	0.1193	0.1041
0.6	0.0987	0.0741	0.0555
0.4	0.0646	0.0359	0.0221
0.2	0.0319	0.0090	0.0048

Approximation

In cases where evaluations of Eq. (1) are not available, various approximations can be made. From symmetry considerations the probability of transmission (a) from a sphere to another surface must be at least as great

as the fraction of 4π steradians subtended by the other surface at the centre of the sphere and (b) from an infinite cylinder, at least as great as the fraction of 2π radians subtended at the axis. Spheres can be approximated to by cubes and circular cylinders by square parallelepipeds, and Eq. (4) can be used to obtain the transmission probabilities. For pairs of identical cylinders or spheres at small separations this procedure over-estimates [1] the probabilities given in Tables II and III.

In arrays, nearer neighbours may partially block the path to more distant neighbours. Various schemas can be used to estimate the resulting reduction in transmission probability. The scheme used here is to extend the array conceptually to infinity and to assume no blockage for successively more distant neighbours (except for those completely blocked from view) until the sum of the probabilities totals unity, after which more distant neighbours are assumed to be totally blocked from view. A reduction of the last probability incorporated in the sum is generally required to make the sum exactly unity.

CRITICALITY CALCULATIONS

There is no restriction on the number of neutron energy groups that may be employed in the calculations. In view of the approximations being made, however, use of many groups can hardly be justified. Where reflectors that alter the energy spectrum are involved there would be some advantage in using at least two groups. In the present treatment a single group is used, i.e. the spatial shape of the flux is assumed to be the same for all energies.

The total current J_i^- (i.e., the current integrated over the surface) received by each surface, i, is given in terms of total transmitted currents J_j^+, and transmission probabilities ρ_{ij}, by

$$J_i^- = \sum_{j=1}^{N} \rho_{ij} \; J_j^+ \qquad i = 1, N \tag{7}$$

where N is the number of surfaces involved. (Unless the surface is concave, ρ_{ii} is zero.)

For a unit to be critical the ratio of J_i^- to J_i^+ must have a definite value depending on the composition and dimensions of the unit. A reasonable approximation [1] is

$$\frac{J^-}{J^+} = \frac{\phi + \frac{2}{3\Sigma} \nabla \phi}{\phi - \frac{2}{3\Sigma} \nabla \phi} \tag{8}$$

where ϕ and $\nabla\phi$ are the flux and its gradient at the surface and Σ is the transport cross-section. It is convenient to express Σ in terms of the bare extrapolation distance, S_0, by making use of the fact that $J^- = 0$ for an

isolated surface and to introduce the critical extrapolation distance, S, to obtain a symmetrical expression for $\beta_i = J_i^- / J_i^+$ (where for a sphere $S = (\pi/B_r) - R$, for a cylinder $S = (2.4048/B_r) - R$, and for a slab $S_1 = (\pi/B_x) - T - S_2$, and where R is the critical radius, T the critical slab thickness, and S_2 the extrapolation distance at the opposite surface. The resulting expressions for slab, cylinder, and sphere are:

$$\beta_i = \frac{\sin B_x(S_1 - S_0)}{\sin B_x(S_1 + S_0)} \tag{9a}$$

$$\beta_i = \frac{\dfrac{J_0(2.4048 - B_r S)}{J_1(2.4048 - B_r S)} - \dfrac{J_0(2.4048 - B_r S_0)}{J_1(2.4048 - B_r S_0)}}{\dfrac{J_0(2.4048 - B_r S)}{J_1(2.4048 - B_r S)} + \dfrac{J_0(2.4048 - B_r S_0)}{J_1(2.4048 - B_r S_0)}} \tag{9b}$$

$$\beta_i = \frac{\dfrac{\pi - B_r S}{1 + (\pi - B_r S)\cot B_r S} - \dfrac{\pi - B_r S_0}{1 + (\pi - B_r S_0)\cot B_r S_0}}{\dfrac{\pi - B_r S}{1 + (\pi - B_r S)\cot B_r S} + \dfrac{\pi - B_r S_0}{1 + (\pi - B_r S_0)\cot B_r S_0}} \tag{9c}$$

For a slab the β_i at its two surfaces are necessarily related so that if β_1 is specified β_2 is given by

$$\beta_2 = \frac{\sin 2B_x(\bar{S} - S_0) - \beta_1 \sin 2B_x\bar{S}}{\sin 2B_x\bar{S} - \beta_1 \sin 2B_x(\bar{S} + S_0)} \tag{10}$$

where $2\bar{S} = \dfrac{\pi}{B_x} - T$.

With the introduction of β_i, Equations (7) become

$$\sum_{j=1}^{N} (\beta_i \delta_{ij} - \rho_{ij}) J_j^+ = 0 \qquad i = 1, N \tag{11}$$

which are homogeneous in the J_j^+; hence a solution is obtained by finding the appropriate values of β_i that make the determinant of the coefficients of the J_i^+ zero. When identical units occupy symmetrically equivalent positions, their J_i^+ are all equal, as are their J_i^-; hence the number of equations can be reduced to the number of symmetrically different positions. If all units are different, the β_i for all units except one can be evaluated from their compositions and sizes by Eq. (9) and the value that β must have for this unit in order that the group be critical is determined from the requirement that the determinant be zero. For a group of identical spheres, cylinders, or slabs treated as though they are symmetrical by setting $\beta_1 = \beta_2 = \dfrac{\sin B_x(\bar{S} - S_0)}{\sin B_x(\bar{S} + S_0)}$, the β_i are all the same; β_i is then the maximum eigenvalue of the matrix of coefficients.

By definition, β_i is the albedo that the medium surrounding a surface must have if the unit is to be critical and $\alpha_i = \beta_i^{-1}$ is the albedo of the surface

as determined by the size and composition of the unit. For reflectors the appropriate albedo may be difficult to calculate. If, however, the extrapolation distance is known when the reflector is in contact with a unit, the albedo of the reflector can be obtained from the relation

$$\alpha_r = \beta_u \tag{12}$$

Although extrapolation distance is insensitive to the radius of curvature of the reflector, the albedo is not, and corrections for changes in curvature should be made. Where a reflector surrounds an array, it is simplest to assume a flat surface and to use Eqs. (9a) and (12) to determine its albedo.

TABLE IV

Average Probability of Transmission, β, to Other Spheres in a Bare Cubic Array

Diameter/Pitch	β in a Cubic Array of: 8	27	64	125
1.0	0.3536	0.5811	0.7132	0.7933
0.8	0.2178	0.4227	0.5732	0.6773
0.6	0.1178	0.2347	0.3503	0.4573
0.4	0.0507	0.1016	0.1523	0.2023
0.2	0.0124	0.0248	0.0373	0.0497

TABLE V

Average Probability of Transmission, β, to Other Cylinders in a Bare Square Array

Diameter/Pitch	Diameter/Ht	β in a Square Array of: 4	9	16	25
1.0	0	0.4317	0.6505	0.7668	0.8343
	0.5	0.4070	0.6085	0.7148	0.7763
	1.0	0.3846	0.5709	0.6684	0.7246
0.8	0	0.3647	0.5853	0.7138	0.7924
	0.5	0.3056	0.4791	0.5771	0.6360
	1.0	0.2568	0.3961	0.4733	0.5193
0.6	0	0.2661	0.4777	0.6214	0.7170
	0.5	0.1882	0.3127	0.3911	0.4413
	1.0	0.1363	0.2202	0.2713	0.3036
0.4	0	0.1744	0.3156	0.4458	0.5530
	0.5	0.0901	0.1485	0.1902	0.2204
	1.0	0.0544	0.0884	0.1118	0.1285
0.2	0	0.0864	0.1566	0.2214	0.2823
	0.5	0.0225	0.0370	0.0472	0.0549
	1.0	0.0118	0.0193	0.0245	0.0285

In Tables IV and V eigenvalues are given for bare arrays of identical spheres and cylinders. For reflected arrays the different locations within the array are affected somewhat differently by the reflector, but a reasonable approximation is to consider the eigenvalue given in the tables as the average probability, ρ, of transmission from a unit to all the other units. The probability of reaching a surrounding reflector is then $1 - \rho$, and the probability of neutrons being transmitted from the reflector to a unit is $(1 - \rho)\, A_u/A_r$.

For a reflected array of N units then

$$\beta = \rho + \frac{(1-\rho)^2 \frac{N A_u}{A_r} \alpha_r}{1 - \alpha_r \left[\frac{1-(1-\rho) N A_u}{A_r}\right]} \tag{13}$$

Comparison of several values of β as calculated from Eq. (13) with the corresponding maximum eigenvalues calculated for reflected cubic arrays of spheres [3] shows that Eq. (13)overestimates β by less than 1%.

In general, β as obtained by Eq. (11) will not equal β as calculated by Eq. (9) from the size and composition of the unit, i.e. S as obtained from Eqs. (9) and (11) will not be consistent with the actual size of the unit. It is convenient to calculate a geometric buckling from this S and the actual dimensions and to calculate k_{eff} as

$$k_{eff} = \frac{1 + M^2 B_m^2}{1 + M^2 B_g^2} \tag{14}$$

where $B_m^2$is the material buckling and M^2 is an appropriate migration area.

By comparing the k_{eff} so calculated for a unit within a group with values calculated for units in similar groups found experimentally to be critical, a judgment can be made as to whether the group would be subcritical. Dimensions, spacings and compositions can be adjusted until k_{eff} has the desired value.

COMPARISON WITH EXPERIMENT

A large number of critical experiments have been performed with groups of interacting units and most of the data have been compiled in a recent publication [4]. Comparison is made here (Tables VI-XVI) in terms of calculated and experimental critical separations (taken in some cases from the original reference rather than from the compilation) and in terms of k_{eff} calculated for the experimentally critical group. The value of k_{eff} is somewhat dependent on the value of M^2 used. A different migration area, M'^2, would give a different k'_{eff} ,

$$k'_{eff} = \frac{1 + \frac{(k_{eff} - 1) M'^2 B_m^2}{k_{eff} + M'^2 B_m^2}}{1 - \frac{(k_{eff} - 1) M'^2}{(k_{eff} + M'^2 B_m^2) M^2}} \tag{15}$$

but for k_{eff} near unity the dependence on M^2 is small.

Calculations are of course dependent on the reactivities assumed for individual units. A careful attempt was made to choose B_m^2 and S_0 consistent with experiments performed with bare isolated units [4]. The choice is not a unique one, however. Once S_0 is selected, B_m^2 is determined; but various S_0 may be chosen. One would hope to choose S_0 such that B_m^2 would

TABLE VI

Parallel 15.1 and 7.6 cm Thick Slabs of Solution Containing 76 g $^{235}U/l$ [4]

$B_m^2 = 0.023306$ cm^{-2}, $M^2 = 32$ cm^2, $S_o = 3.0$ cm

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
1 - ½	24.9	0.6	0.4	0.997
	32.4	5.7	7.1	1.010
	44.6	15.9	20.9	1.017
	58.6	31.1	39.8	1.014
	65.8	38.7	50.7	1.014
	86.7	76.8	88.6	1.004
	92.5	76.8	99.8	1.008
	113.8	107.3	163.2	1.007
½ - 1 - ½	19.6	0.6	0.1	0.994
	44.2	26.0	32.4	1.019
	62.5	51.4	63.9	1.015
	81.5	81.9	102.1	1.010
1 - 1	25.4	5.7	8.0(8.2)[a]	1.016(1.021)[a]
	32.8	15.9	21.5(21.3)	1.023(1.024)
	44.9	38.7	51.4(50.7)	1.020(1.020)
	50.3	51.4	67.5(66.5)	1.018(1.017)
	59.7	76.8	99.6(98.1)	1.014(1.013)
	73.2	122.6	155.5(151.8)	1.009(1.008)
	82.1	168.3	201.9(201.9)	1.005(1.005)
½ - ½ - ½	25.5	0.6	0.5(0.8)[a]	0.998(1.001)[a]
	34.4	3.2	3.8(3.8)	1.006(1.007)
	58.8	8.3	10.6(9.9)	1.016(1.014)
	85.5	12.1	15.7(14.4)	1.020(1.015)
	107.4	14.6	18.8(17.1)	1.020(1.015)
	120.4	15.9	20.3(18.4)	1.020(1.014)

(a) Values in parentheses calculated with $S_o = 2.5$ cm, $B_m^2 = 0.025585$ cm^{-2}

TABLE VII

Parallel 7.6 cm Thick Slabs of Solution Containing 480 g $^{235}U/l$ [4]

$B_m^2 = 0.031330$ cm^{-2}, $M^2 = 32$ cm^2, $S_o = 2.5$ cm

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
½ - ½	33.4	0.9	0	0.988
	44.5	3.2	2.3	0.992
	59.4	5.7	4.8	0.993
	~67.0	7.0	5.8	0.992

be independent of shape, but this is probably an impossible goal. The dependence on S_0 is fairly small, however, as can be seen from Tables VI and XV.

In calculations for solutions the actual separations between solutions were used and vessel walls were ignored (i.e. treated as vacuum). Groups involving thick and thin slabs were assumed to contain slabs of a particular thickness and slabs of exactly half this thickness. No comparisons of calculations and experiments with three dimensional arrays of units are included since such comparisons have recently been published [2,3]. Not all the data on pairs of cylinders are included since results do not differ significantly from those reported here. The cylinders of solution in Table XIV were all assumed to have diameters of 15.24 cm although in the experiments

TABLE VIII

Perpendicular 15.1 and 7.6 cm Thick Slabs of Solution Containing 79 g $^{235}U/l$ [4]

$B_m^2 = 0.023438\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 3.0$ cm

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
1 - 1 (T)	45.2	3.4	4.9	1.003
	53.8	9.1	20.1	1.015
	67.4	24.4	48.6	1.017
	82.8	47.2	89.7	1.014
	84.1	52.3	94.3	1.013
	87.8	62.5	107.8	1.010
½ - 1 (T)	79.3	9.1	6.0	0.998
	94.9	24.4	24.4	1.000
	105.7	47.2	47.2	1.000
1 - 1 (L)	56.9	1.5	25.7	1.020
	71.3	16.8	90.0	1.026
	76.6	29.5	129.6	1.023
	79.6	37.1	168.7	1.022
	84.6	57.4	285.3	1.018
	88.9	77.7	-	1.015
½ - 1 (L)	97.9	1.5	28.3	1.006
	102.4	16.8	40.8	1.004

TABLE IX

Parallel Slabs of Uranium Metal [4]

$B_m^2 = 0.08258$, $M^2 = 15.7$, $S_o = 2.1$ cm

Surface Dimensions, cm	Thickness, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
20.3 x 25.4 (Two slabs)	4.76	0.3	0.5	1.003
	5.08	1.0	1.4	1.008
	5.40	1.6	2.4	1.014
	5.71	2.4	3.4	1.018
	6.03	3.2	4.6	1.022
	6.67	5.2	7.2	1.026
	6.98	6.4	8.8	1.027
	7.30	8.0	10.6	1.026
	7.62	9.9	12.8	1.024
	7.94	12.5	15.7	1.020
38.1 (dia) (Two slabs)	4.15	1.2	1.9	1.009
	4.46	2.5	3.8	1.017
	4.77	3.9	5.9	1.023
	5.08	5.6	8.0	1.028
	5.38	7.4	10.5	1.032
	5.72	9.6	13.5	1.035
	6.00	12.3	16.3	1.032
	6.34	15.7	20.6	1.031
27.94 (dia) (Two slabs)	4.46	0.3	0.6	1.005
	4.77	1.2	1.8	1.011
	5.08	1.9	3.1	1.018
	5.38	2.9	4.3	1.022
	5.74	3.9	5.9	1.029
	6.00	5.1	7.2	1.028
	6.34	6.5	9.1	1.031
	6.69	8.2	11.3	1.032
	7.00	10.2	13.7	1.030
	7.31	12.9	16.7	1.025
17.78 (dia) (Two slabs)	6.69	0.3	0.5	1.004
	6.95	0.6	0.9	1.005
	7.31	0.9	1.4	1.010
	7.61	1.2	1.8	1.012
	7.92	1.5	2.2	1.013
	8.28	1.9	2.8	1.016
38.1 (dia) (Three slabs)	3.17	2.2	3.3	1.017
	3.81	4.5	6.6	1.032
	4.44	7.2	10.3	1.042
	4.75	8.8	12.3	1.043
	5.08	10.7	14.6	1.045

TABLE X

15.1 cm Thick Slab of Solution
Containing 79 g ^{235}U/l Parallel to a Reflector [5,6]

$B_m^2 = 0.023451$, $M^2 = 32$, $S_o = 3.0$

Reflector Albedo	Ht, cm	Separation, cm Exptl.	Calc.(1)(a)	Calc.(9)(b)	$k_{eff}(1)$(a)	$k_{eff}(9)$(b)
0.475 (concrete)	32.3	0	0	0	1.000	1.000
	47.8	15.2	13.2	14.1	0.995	0.997
	69.6	45.7	40.5	47.6	0.997	1.001
	80.3	68.6	58.7	72.8	0.997	1.001
	92.2	106.7	83.5	114.3	0.997	1.001
	107.2	228.6	-	228.6	0.999	1.000
0.154 (1.27 cm thick steel)	55.3	0	0		1.000	
	63.2	7.6	8.2		1.001	
	70.5	15.2	17.7		1.002	
	81.3	30.5	36.6		1.002	
	92.3	61.0	73.2		1.001	
	97.1	91.4	109.7		1.001	

(a) For (1), reflector surface dimensions assumed the same as those of solution surface;
(b) For (9), reflector assumed to consist of 9 such rectangles forming a rectangle 3H x 3L; neutrons reflected to the ends were ignored.

TABLE XI

Groups of Cylinders of Solution
Containing about 500 g ^{235}U/l [4]

$B_m^2 = 0.030367$ cm^{-2}, $M^2 = 32$ cm^2, $S_o = 2.7$ cm
L = Linear, T = Triangular, S = Square

Dia, cm	Config-uration	Ht, cm	Pitch, cm Exptl.	Calc.	k_{eff}
12.7	7-T	28.7	13.6	12.7(a)	0.924
		66.3	15.9	14.1	0.947
15.2	7-T	22.6	15.9	15.2(a)	0.935
		33.0	18.1	16.6	0.958
		51.6	20.6	19.1	0.973
		83.8	23.2	21.7	0.981
20.3	2-L	68.3	21.0	20.3(a)	0.979
	3-L	45.7	21.0	20.3(a)	0.981
		124.5	28.2	26.9	0.995
	4-L	41.9	21.0	20.3(a)	0.988
		96.5	28.2	28.0	0.999
	5-L	40.1	21.0	20.4	0.992
		78.7	28.2	27.6	0.996
20.3	3-T	27.2	21.0	20.3(a)	0.938
		35.1	23.2	20.3(a)	0.967
		45.2	25.7	23.5	0.982
		55.9	28.2	26.3	0.988
		68.8	30.8	28.9	0.991
		106.7	35.9	33.9	0.993
	3-T (120°)	42.4	21.0	20.3(a)	0.969
		87.4	28.2	26.1	0.993
	3-S	36.1	21.0	20.3(a)	0.967
		71.4	28.2	26.8	0.992
20.3	7-T	18.3	21.0	20.3(a)	0.943
		21.6	23.2	21.1	0.962
		25.7	25.7	24.1	0.977
		29.7	28.2	27.1	0.988
		33.5	30.8	30.1	0.994
		41.9	35.9	35.4	0.997
		55.9	43.5	42.7	0.996
25.4	2-L	25.5	26.0	25.4(a)	0.977
		29.9	35.7	31.8	0.993
		32.6	55.8	47.8	0.996
		33.6	∞		0.992

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE XII

Groups of Cylinders of Solution Containing 84 g ^{235}U/l [4]

B_m^2 = 0.024384 cm^{-2}, M^2 = 32 cm^2, S_o = 3.0 cm

Dia, cm	Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
15.2	7-T	31.0	16.3	15.2[a]	0.947
		56.9	18.1	17.2	0.978
		195.6	20.6	20.0	0.991
20.3	3-T	41.4	21.0	20.3[a]	0.945
		79.3	23.2	20.3[a]	0.978
20.3	7-T	28.7	23.2	22.2	0.982
		45.2	28.2	28.8	1.006
		89.9	35.9	37.9	1.015
		119.1	38.4	40.6	1.014

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE XIII

Groups of 24.1 cm Diameter Cylinders of Solution Containing 87 g ^{235}U/l [4]

B_m^2 = 0.024483 cm^{-2}, M^2 = 32 cm^2, S_o = 3.0 cm
L = Linear, T = Triangular, S = Square

Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
2-L	61.2	27.0	26.4	0.998
	80.5	32.1	32.8	1.002
	113.0	39.7	42.1	1.003
	137.2	44.8	47.5	1.002
3-L	56.6	29.5	30.8	1.005
	84.3	39.7	43.4	1.006
	111.3	49.8	53.7	1.004
	152.7	62.5	66.3	1.002
4-L	57.7	32.1	34.1	1.007
	76.2	39.7	43.6	1.007
	97.8	49.8	53.6	1.004
5-L	55.1	32.1	34.2	1.008
	71.9	39.7	43.4	1.007
	92.0	49.8	53.5	1.004
6-L	54.1	32.1	34.5	1.009
	88.4	49.8	53.3	1.004
2 x 2 - S	40.1	32.1	32.4	1.002
	69.1	49.8	53.7	1.007
	120.4	80.3	85.9	1.003
	158.8	100.6	104.0	1.001
3-T	34.0	27.0	24.1[a]	0.981
	51.6	34.6	35.2	1.002
	71.4	44.8	47.4	1.005
	92.2	54.9	59.1	1.004
	126.2	70.2	74.9	1.003
	152.7	80.3	84.7	1.002
7-T	30.7	32.1	32.7	1.005
	51.1	49.8	53.7	1.011
	83.6	80.3	85.9	1.005

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE XIV

Groups of 15.24 cm Diameter Cylinders of Solution Containing 380 g ^{235}U/l [4,7]

B_m^2 = 0.026822 cm^{-2}, M^2 = 35 cm^2, S = 3.1 cm
T = Triangular, S = Square

Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
2 x 2 - S	66.9	15.9	15.2[a]	0.942
	108.7	16.5	15.2[a]	0.954
3 x 3 - S	51.5	19.4	19.6	1.006
	55.0	19.9	20.0	1.003
	77.9	21.4	21.8	1.007
	101.6	22.4	23.0	1.009
	126.0	23.2	23.8	1.009
4 x 4 - S	50.1	22.1	23.4	1.025
	76.2	25.1	27.2	1.035
	101.8	27.0	29.2	1.035
	128.0	28.2	30.6	1.033
7-T	24.8	15.9	15.2[a]	0.948
	39.1	18.1	17.2	0.978
	69.8	20.6	20.2	0.994
	99.5	21.9	21.8	0.999
19-T	50.1	24.5	26.5	1.041
	76.2	28.1	30.8	1.038
	101.6	30.6	33.7	1.035
	127.0	32.4	35.4	1.035

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE XV

Groups of 24.13 cm Diameter Cylinders of Solution Containing about 890 g U(4.9)/l [8]

B_m^2 = 0.011970 cm^{-2}, M^2 = 31 cm^2, S_o = 3.00 cm
T = Triangular, S = Square

Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
3 x 3 - S	61.0	26.1	24.1[a]	0.964(0.942)[b]
	122.0	29.6	27.3(24.3)[b]	0.978(0.955)
	142.2	30.3	28.1(24.9)	0.977(0.954)
4 x 4 - S	61.0	28.3	28.2(26.3)	0.998(0.979)
	122.0	33.9	33.9(31.6)	1.002(0.979)
	142.2	35.1	35.1(32.5)	1.000(0.979)
5 x 5 - S	61.0	30.0	30.7(29.4)	1.008(0.991)
	122.0	37.1	39.5(36.5)	1.015(0.996)
	142.2	38.9	41.3(38.2)	1.015(0.996)
7-T	61.0	26.6	24.1[a]	0.949(0.926)
	122.0	29.2	25.4(24.1)[a]	0.962(0.939)
	142.2	29.8	25.9(24.1)[a]	0.963(0.939)
19-T	122.0	37.9	39.4(37.2)	1.013(0.994)
	142.2	39.5	40.7(38.4)	1.010(0.989)

(a) Calculations indicated cylinders would be subcritical at contact.
(b) Values in parentheses calculated with S_o = 2.50 cm, B_m^2 = 0.012536.

this was the outer diamter of some of the containers. In the calculations for the interaction between a slab and a cylinder (Table XVI), the cylinder was treated as a square cylinder having the same volume in computing the probability of transmission to the slab; this probability was multiplied by the

TABLE XVI

Cylinder (25.4 cm dia) and
Slab (15.1 cm thick, 120.6 cm long)
of Solution Containing 78-g $^{235}U/l$ [4]

$B_m^2 = 0.024793\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 2.70\ cm$

Ht, cm	Separation, cm	ρ_{sc}	ρ_{cs}	β_c	β_s	k_{eff}
30.7	0.7	0.355	0.235	0.360	0.231	0.934
47.8	15.7	0.246	0.163	0.169	0.237	1.001
58.2	31.0	0.183	0.121	0.113	0.196	1.010
66.5	46.2	0.140	0.093	0.085	0.153	1.010
79.3	76.7	0.087	0.058	0.056	0.089	1.006
89.2	107.2	0.059	0.039	0.039	0.058	1.004
115.8	∞	Isolated slab				1.000
147.3	∞	Isolated cylinder				1.000

ratio of the actual slab and cylinder areas in computing the probability of transmission from slab to cylinder.

CONCLUSIONS

In general, agreement between calculations and experiment is quite satisfactory for nuclear safety work. For cylinders the trend towards low k_{eff} values and toward underestimates of critical separations at small spacings is expected and is easily allowed for. The trend toward high k_{eff} values for large groups probably results in part from the conservative manner in which account is taken of partial shielding of units and again can be allowed for. It must be emphasized that accurate or conservative estimates of the reactivities of individual units are required in any application of this method.

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DISCUSSION

P.B. SUHR: You say that this method of calculating interaction can be extended to more than one energy group. However, it strikes me that

even with only two energy groups a solution of the equations set out in formula (7) of your paper would become much more complicated as a result of the coupling between the energy groups. Could you tell us how you would calculate this coupling? Would you use ordinary two-group constants (slowing-down cross-sections, resonance escape probabilities and so on)?

H. K. CLARK: I would use ordinary two-group constants but would adjust them in such a manner as to give agreement with critical experiments performed on bare, isolated units. It is, of course, perfectly true that the extension of the method to more than one energy group makes it more complicated. This problem is described in Ref. [1] of the paper.