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Application of a Simple, Practical Method for Computing Interaction to Arrays Found Experimentally to be Critical*

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Calculations of the critical sizes of cubic arrays of interacting fissionable units are compared with critical experiments. The units are of two types: vessels containing 5 liters of an aqueous solution of highly enriched uranium, and cylinders of highly enriched uranium metal. The arrays are surrounded by various thicknesses of hydrogenous reflectors. Agreement between calculation and experiment is reasonably good. The similarity of the results obtained with the widely differing types of units invites confidence in general applications of the method of calculation. Tables are presented for computing critical and safe sizes of cubic arrays of 8, 27, 64, or 125 identical units as a function of the albedo of the reflector surrounding the array and of the reactivity of an individual unit.

INTRODUCTION

In a recent paper[1], a simple, practical method was described for computing the interaction in groups of fissionable units. Application of this method to critical experiments performed with aqueous solutions of U^{235} confined within pairs of parallel slabs, within groups of three parallel slabs, within pairs of perpendicular slabs, within groups of cylinders containing from two to seven cylinders, and within a slab parallel to a reflecting concrete wall has shown it to be reasonably accurate and generally conservative in the sense that critical systems are generally calculated to be somewhat supercritical[2]. When the paper was written, however, the only large arrays studied experimentally to which the method could be applied were cubic arrays of composite metal spheres[3] for which overall source multiplications were measured as the arrays were built toward 27 units (3 × 3 × 3). Recently, critical experiments have been performed by Thomas[5-7] with essentially cubic arrays of both solution and metal units. The arrays ranged from 8 to 125 units in size, and were surrounded by various thicknesses of hydrogenous reflectors. In the present paper, the method for calculating interaction described in Ref. 1 is applied to these experiments.

APPROXIMATIONS

Several approximations are made to simplify the calculations:

(1) The same spatial distribution is assumed within a unit for neutrons of all energies (one-group approximation) and this distribution is assumed to satisfy the wave equation.

(2) The emitted and incident neutron currents j^+ and j^- are treated as though they were uniform over the entire surface of each unit (including any

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[1] H. K. CLARK, *Nucl. Sci. Eng.* 15, 20-28 (1963).

[2] H. K. CLARK, "Interaction of Subcritical Components," DP-312 (1958).

[3] E. C. MALLORY, H. C. PAXTON and R. H. WHITE, "Safety Tests for the Storage of Fissile Units," LA-1875 (declassified with deletions April 1958).

[4] H. K. CLARK, "Handbook of Nuclear Safety," DP-532 (1961).

[5] J. T. THOMAS, *Trans. Am. Nucl. Soc.* 6 (1), 169-170 (1963).

[6] J. T. THOMAS, "Critical Three-Dimensional Arrays of Neutron-Interacting Units," ORNL-TM-719 (1963).

[7] J. T. THOMAS, "Critical Arrays of U(93.2) Metal Cylinders," pp. 58-62, ORNL-3499 (1963).

surrounding reflector). This approximation tends to underestimate the interaction, particularly in small bare arrays of closely spaced units.

(3) The angular distribution of emitted neutrons is taken to be proportional to the cosine of the angle between the direction of emission and the normal to the element of surface through which the neutrons are emitted.

(4) In arrays of units, complete shielding (such as occurs when units are directly in line) is allowed for, but partial shielding of units by intermediate units is ignored until the fractions of the neutrons reaching successively more distant units in an infinite array formed by extending the finite array in question total unity, after which all more distant units are taken to be completely blocked from view. This approximation tends to overestimate the interaction in varying degrees depending on the size of the array and the spacings within the array.

(5) The neutron currents leaving and entering the surface of a unit are assumed to be given by Eq. 5[1].

(6) The boundary condition that the incoming current j^- is zero for a bare isolated unit is employed to express Σ_{tr} in terms of a bare extrapolation distance S_0 that is consistent with the bare critical size, as inferred from experiment, of a unit of the same composition as a unit in the array and with a calculated value of the material buckling B^2.

(7) The actual surface area of the reflector is employed in the calculations, and the reflector is assigned an albedo characteristic of an infinite slab reflecting an infinite slab of fissionable material having the same composition as a unit in the array. This albedo is β as calculated from Eq. 20[1] with bare and reflected extrapolation distances S_0 and S consistent with critical bare and reflected slab thicknesses, as inferred from experiment, and with the calculated value of the material buckling referred to above.

(8) The cylindrical units used in the experiments are approximated by spheres having the same volume. The expression for the fraction of the neutrons emitted by one sphere that reaches an identical sphere (Eq. 8[1]) was reduced to a double integral which was evaluated on an IBM 704 by Gauss quadrature with the results given in Table I.

(9) In the experiments a common critical surface-to-surface separation rather than a common center-to-center separation was determined. To simplify the calculations the cell around each unit was approximated by a cube having the same volume; hence, each

TABLE I

Fraction of Neutrons Emitted by a Sphere That Reaches an Identical Sphere

Ratio of diameter to center-to-center separation	Fraction
1.0	0.07820
0.95	0.06841
0.90	0.06020
0.85	0.05281
0.80	0.04606
0.75	0.03989
0.70	0.03434
0.65	0.02929
0.60	0.02470
0.55	0.02054
0.50	0.01681
0.45	0.01349
0.40	0.01057
0.35	0.00803
0.30	0.00586
0.25	0.00404
0.20	0.00257
0.15	0.00143
0.10	0.00063
0.05	0.00016

between adjacent units in all three directions. The reflector, which in an actual array was separated by half the surface-to-surface separation from adjacent units, was assumed to be separated by half the center-to-center separation from the plane on which the centers of adjacent units lay.

(10) All structural members present in the experiments to hold the units in place were ignored. In the solution experiments the tubing by which the central units were filled was likewise ignored.

A code was written in FORTRAN for the IBM 704, based on the foregoing approximations, to compute the maximum eigenvalue β of the homogeneous set of equations formed by replacing J^- by βJ^+ in Eq. 15[1] for cubic arrays of spheres as a function of the ratio of sphere diameter to lattice pitch and of the albedo of the reflector. Results obtained for arrays of 8, 27, 64, and 125 units at the albedos chosen to represent the reflectors employed in the experiments are given in Tables II-V.

SOLUTION EXPERIMENTS[5,6]

Three solution concentrations were used in these experiments: 415, 279, and 63.3 g uranium per liter. The uranium contained 92.6% U^{235}. In addition to the approximations outlined above,

TABLE II

Albedo of the Environment of a Spherical Unit in a 2 × 2 × 2 Cubic Array

Diameter/pitch	Albedo of reflector									
	0	0.175	0.206	0.332	0.416	0.447	0.528	0.550	0.660	0.719
1.0	0.3536	0.4348	0.4502	0.5163	0.5639	0.5822	0.6321	0.6463	0.7207	0.7634
0.9	0.2828	0.3648	0.3806	0.4493	0.4996	0.5192	0.5732	0.5887	0.6711	0.7195
0.8	0.2178	0.2961	0.3115	0.3795	0.4305	0.4506	0.5069	0.5233	0.6123	0.6659
0.7	0.1632	0.2330	0.2471	0.3104	0.3592	0.3788	0.4347	0.4512	0.5436	0.6013
0.6	0.1178	0.1759	0.1879	0.2429	0.2868	0.3047	0.3570	0.3728	0.4639	0.5234
0.5	0.0805	0.1251	0.1345	0.1787	0.2151	0.2302	0.2755	0.2895	0.3733	0.4310
0.4	0.0507	0.0817	0.0884	0.1203	0.1473	0.1589	0.1941	0.2052	0.2747	0.3253
0.3	0.0282	0.0467	0.0507	0.0705	0.0877	0.0952	0.1185	0.1260	0.1749	0.2127
0.2	0.0124	0.0210	0.0228	0.0323	0.0406	9.0443	0.0560	0.0599	0.0858	0.1069
0.1	0.0031	0.0053	0.0057	0.0082	0.0104	0.0114	0.0146	0.0156	0.0229	0.0290

TABLE III

Albedo of the Environment of a Spherical Unit in a 3 × 3 × 3 Cubic Array

Diameter/pitch	Albedo of reflector								
	0	0.175	0.206	0.332	0.416	0.447	0.550	0.660	0.719
1.0	0.5811	0.6293	0.6387	0.6800	0.7104	0.7222	0.7641	0.8133	0.8418
0.9	0.5032	0.5591	0.5700	0.6181	0.6536	0.6674	0.7164	0.7744	0.8083
0.8	0.4227	0.4838	0.4958	0.5489	0.5885	0.6040	0.6595	0.7265	0.7661
0.7	0.3243	0.3895	0.4025	0.4604	0.5042	0.5216	0.5845	0.6620	0.7087
0.6	0.2347	0.2972	0.3099	0.3676	0.4124	0.4305	0.4972	0.5823	0.6354
0.5	0.1607	0.2140	0.2251	0.2767	0.3180	0.3350	0.3996	0.4864	0.5432
0.4	0.1016	0.1415	0.1500	0.1905	0.2243	0.2384	0.2941	0.3738	0.4292
0.3	0.0565	0.0817	0.0872	0.1141	0.1372	0.1471	0.1875	0.2493	0.2955
0.2	0.0248	0.0370	0.0397	0.0531	0.0650	0.0702	0.0921	0.1278	0.1563
0.1	0.0061	0.0093	0.0101	0.0137	0.0169	0.0184	0.0246	0.0352	0.0441

TABLE IV

Albedo of the Environment of a Spherical Unit in a 4 × 4 × 4 Cubic Array

Diameter/pitch	Albedo of reflector		
	0	0.550	0.719
1.0	0.7132	0.8293	0.8836
0.9	0.6466	0.7908	0.8573
0.8	0.5732	0.7437	0.8236
0.7	0.4685	0.6769	0.7756
0.6	0.3503	0.5903	0.7096
0.5	0.2406	0.4858	0.6215
0.4	0.1523	0.3680	0.5071
0.3	0.0848	0.2421	0.3634
0.2	0.0373	0.1225	0.2009
0.1	0.0092	0.0334	0.0588

TABLE V

Albedo of the Environment of a Spherical Unit in a 5 × 5 × 5 Cubic Array

Diameter/pitch	Albedo of reflector		
	0	0.550	0.719
1.0	0.7933	0.8700	0.9096
0.9	0.7392	0.8384	0.8881
0.8	0.6773	0.7995	0.8606
0.7	0.5795	0.7408	0.8197
0.6	0.4573	0.6617	0.7622
0.5	0.3193	0.5555	0.6795
0.4	0.2023	0.4301	0.5676
0.3	0.1128	0.2906	0.4198
0.2	0.0497	0.1511	0.2411
0.1	0.0123	0.0421	0.0731

spheres enclosed by 1/4-inch-thick Plexiglas[a] and hence were assumed to have an outer diameter of 8.85 inches. The Plexiglas was assumed to be equivalent to the same volume of H_2O and was homogenized with the solution to give effective concentrations of 349, 234, and 53.2 g of uranium per liter. The material bucklings of these homogenized solutions were calculated to be 0.0306, 0.0294, and 0.0191 cm^{-2}, values which with respective bare extrapolation distances of 2.6, 2.8, and 3.5 cm are consistent with experimental data for similar solutions[4]. The respective migration areas, required for obtaining k_{eff}, were calculated to be 32.3, 32.4, and 33.6 cm^2. The Plexiglas and paraffin reflectors were assumed to be equivalent to the same thickness of H_2O. The albedo as a function of reflector thickness was estimated[4] from experimental data to have the values in Table VI for the 349 and 234 g/liter solutions. The albedo of 0.55 corresponds to extrapolation distances of 5.9 and 6.2 cm, respectively, for these two solutions.

The experiments with the 415 g/liter solution for which calculations were made are summarized in Table VII together with the corresponding values of the eigenvalue β computed by the IBM 704. The albedo β, which the environment of a spherical unit of radius R must have if it is to be critical, was calculated by Eq. 21[1] from the composition and diameter of a unit (i.e. with $B^2 = 0.0306$ cm^{-2}, $S_0 = 2.6$ cm, and $S = \frac{\pi}{B} - 1.27 \times 8.85 = 6.72$ cm) to be 0.446. Besides comparing this value of β with those of Table VII, the comparisons that will be made here consist (1) of comparing the critical separations of Table VII with those that make the eigenvalue $B = 0.446$, and (2) of solving Eq. 21[1] for the values of S that give the values of β in Table VII and using these values of S to compute values of

$$k_{eff} = \frac{1 + M^2 B^2}{1 + M^2 \frac{\pi^2}{(R+S)^2}} \qquad (1)$$

relating theory and experiment. These two comparisons of theory and experiment are shown in Table VIII.

Only a few experiments were performed with the solutions of lower concentration. An 8-unit reflected array of 279 g/liter units was found experimentally to have a critical surface-to-surface spacing of 8.71 cm. With a reflector

TABLE VI

Albedo of Water Reflector
as a Function of Thickness

Thickness (inches)	Albedo
0.5	0.206
1.5	0.416
3.0	0.528
6.0	0.550

TABLE VII

Calculated Values of the Critical Albedo β
Provided for a 415 g/l Solution Unit by the Other
Units and by the Reflector Surrounding the Array

Number of units	Reflector thickness (cm)	Surface-to-surface separation (cm)	Diameter/avg pitch	Critical albedo β
8	0	1.43	$>1^a$	-
	1.27[b]	3.00	0.9824	0.4394
	1.27	3.28	0.9705	0.4324
	3.81	6.91	0.8390	0.4578
	7.62	8.48	0.7926	0.5018
	15.24	8.99	0.7786	0.5084
27	0	6.48	0.8527	0.4662
	1.27[b]	8.76	0.7848	0.4833
	1.27	9.02	0.7778	0.4769
	3.81	13.69	0.6696	0.4767
	15.24	16.53	0.6174	0.5130
64	0	10.67	0.7358	0.5077
125	0	14.40	0.6557	0.5291

[a] The equivalent spheres are in contact when the surface-to-surface separation is 2.5 cm; hence no calculation of interaction was made for this case.

[b] This reflector was Plexiglas. The other reflectors were paraffin.

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[a] Trademark of Rohm and Haas Co.

TABLE VIII

Comparisons of Calculations and Experiments with 415 g/l Solution Units

Number of units	Reflector thickness (cm)	Avg pitch (cm)		k_{eff}
		Exptl.	Calc.	
8	0	<22.48	-	-
	1.27[a]	22.88	22.7	0.995
	1.27	23.16	22.7	0.990
	3.81	26.79	27.3	1.009
	7.62	28.36	31.4	1.046
	15.24	28.87	32.3	1.051
27	0	26.36	27.1	1.016
	1.27[a]	28.64	30.3	1.030
	1.27	28.90	30.3	1.025
	3.81	33.57	35.3	1.025
	15.24	36.41	41.0	1.055
64	0	30.55	33.3	1.051
125	0	34.28	38.0	1.069

[a] Plexiglas reflector.

albedo of 0.55, the albedo of the environment of a unit was calculated to be 0.5137. By Eqs. 21[1] and 1, k_{eff} was calculated to be 1.059, which is close to the value calculated for the reflected 27-unit array of 415 g/liter units. A 27-unit bare array of 279 g/liter units had a critical surface-to-surface spacing of 6.40 cm. The albedo of the environment of a unit was calculated to be 0.4684, and k_{eff} was calculated to be 1.021, which may be compared with 1.016, the value calculated for the corresponding array of 415 g/liter units. The critical surface-to-surface spacing was found to be 2.41 cm for the bare 27-unit array of 63.3 g/liter units. Since this array is only slightly more compact than a cubic array of spheres in contact, Table III was extrapolated slightly to obtain the albedo of the environment, 0.588. The value of k_{eff} calculated from Eqs. 21[1] and 1 is 1.015.

In general, agreement between calculations and experiment is reasonably good. There are trends toward increased k_{eff} as the size of the bare array increases and as the thickness of the reflector increases. From the practical point of view of computing safe arrays it is gratifying that the calculations are for the most part conservative. Some of this conservatism, however, may stem from homogenizing the Plexiglas walls in with the solution. Two-group calculations indicate that the critical albedo at the outer surface is about 7% larger than that for the homogenized sphere. The result of this increase is essentially a displacement of the relation between theory and experiment roughly equivalent to subtracting 0.025 from each k_{eff} in the last column of Table VIII.

EXPERIMENTS WITH URANIUM CYLINDERS[5,7]

The smaller cylinders were approximated by spheres having diameters of 12.88 cm and the larger cylinders by spheres having diameters of 13.88 cm. The material buckling of the uranium (18.66 g/cm^3, 93.2% U^{235}) was calculated[4] to be 0.08204 cm^{-2}, which together with a bare extrapolation distance of 2.17 cm is consistent with the experimentally determined bare critical mass of a sphere. The migration area was calculated[4] to be 15.7 cm^2. The paraffin reflectors were assumed to be equivalent to Plexiglas of the same thickness since Tables VII and VIII indicate this to be a fairly good approximation and since data[8] are available giving the effectiveness of Plexiglas as a reflector for uranium (18.7 g/cm^3, 93.4% U^{235}) slabs as a function of reflector thickness. The albedo as a function of reflector thickness for infinite slabs is given in Table IX. These albedos were calculated by Eq. 20[1] from the calculated material buckling of 0.08204 cm^{-2} and from extrapolations to zero transverse (radial) geometric buckling of average extrapolation distances obtained by equating a calculated material buckling of 0.08258 cm^{-2} to the geometric bucklings of the slabs studied experimentally. The extrapolation distances obtained for the infinite bare slab and for the infinite slab reflected by 15.24 cm of Plexiglas were, respectively, 2.26 and 4.74 cm. For a critical cylinder having its height equal to its diameter the corresponding average extrapolation distances indicated by the experiments are 2.11 and 4.07 cm. For a sphere surrounded by a paraffin reflector[4] the extrapolation distance is about 4.20 cm. The effect of shape on the average extrapolation distance for the reflected metal is much larger than for solution units.

TABLE IX

Albedo of Plexiglas in Contact with an Infinite Slab of Uranium (93.2% U^{235}) as a Function of Thickness

Thickness of Plexiglas reflector (cm)	Albedo
0	0
1.27	0.175
2.54	0.332
3.81	0.447
7.62	0.660
15.24	0.719

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[8] J. T. MIHALCZO and J. J. LYNN, "Critical Parameters of Bare and Reflected 93.4 wt% U^{235}-Enriched Uranium Metal Slabs," pp. 73-76, ORNL-3016 (1960).

In Table X the experiments performed with the arrays of metal units are summarized, and values of the eigenvalue β computed by the IBM 704 are given. Solution of Equation 21[1] for β gives values of 0.371 and 0.309, respectively, for the smaller and larger units. Comparison between calculations and experiments is made in terms of average center-to-center spacing and of k_{eff} in Table XI. As in the case of the solution experiments, for bare arrays k_{eff} increases with the number of units, and k_{eff} for arrays surrounded by a thick reflector is much larger than for bare arrays.

EFFECT OF INTERSPERSED MATERIALS

In many practical cases materials may be interspersed within an array. Additional experiments were performed[7] in which the smaller metal cylinders were centered in Plexiglas boxes, in sections of steel pipe, and in sections of pipe within Plexiglas boxes. The introduction of these materials complicates the calculations, but it can be accounted for in an approximate manner by assuming that material of the same thickness fits tightly around each unit so that the equivalent 12.88-cm-diameter spheres, for the boxes, are

TABLE XI

Comparisons of Calculations and Experiments with Metal Cylinders

Number of Units	Size	Reflector thickness (cm)	Avg pitch (cm)		k_{eff}
			Exptl.	Calc.	
8	Small	0	13.47	12.6[a]	0.960
		0	13.50	12.6[a]	0.959
		1.27	14.93	14.1	0.972
		2.54	16.96	16.3	0.982
		3.81	19.46	18.7	0.982
		7.62	22.76	25.9	1.056
		15.24	23.24	29.2	1.102
27	Small	0	17.61	17.3	0.987
		1.27	19.80	19.0	0.975
		3.81	26.01	23.9	0.965
		7.62	29.97	32.3	1.034
		15.24	30.40	36.4	1.079
8	Large	0	15.68	14.8	0.971
		1 27	17.56	16.9	0.984
		3.81	23.67	22.9	0.988
		7.62	27.84	32.1	1.051
		15.24	28.52	36.4	1.091
27	Large	0	20.63	20.4	0.991
		1.27	23.47	22.7	0.984
		3 81	31.75	29.2	0.972

[a] Corresponds to a Diameter/Pitch ratio >1 and was obtained by extrapolation.

TABLE X

Calculated Values of the Critical Albedo β Provided for a Metal Unit by the Other Units and by the Reflector Surrounding the Array

Number of units	Size of unit	Reflector thickness (cm)	Surface-to-surface separation (cm)	Diameter/ avg pitch	Critical albedo β
8	Small	0[a]	2.217	0.9563	0.324
		0	2.248	0.9541	0.323
		1.27	3.678	0.8627	0.338
		2.54	5.710	0.7594	0.351
		3.81	8.207	0.6619	0.351
		7.62	11.509	0.5659	0.434
		15.24	11.986	0.5543	0.483
27	Small	0	6.363	0.7312	0.356
		1.27	8.547	0.6506	0 343
		3.81	14.764	0.4951	0.330
		7.62	18.720	0.4298	0 409
		15.24	19.147	0.4237	0.458
8	Large	0	3.543	0.8854	0 273
		1 27	5.423	0.7905	0.290
		3.81	11 532	0.5863	0.294
		7.62	15.697	0.4985	0 372
		15.24	16.378	0.4866	0.418
27	Large	0	8.494	0.6727	0.298
		1.27	11.323	0.5915	0.290
		3.81	19.606	0.4371	0.274

[a] The average unit weight was 20.805 kg, compared with 20.960 kg for the other 8-unit arrays of small cylinders and 20.877 kg for the 27-unit arrays

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enclosed by spherical shells with outer diameters of 14.15 and 15.42 cm. The critical albedos or eigenvalues β_2 at the outer surfaces of these shells were calculated on the IBM 704 in the same manner as for bare units. Although the albedo of the reflector undoubtedly is influenced by moderation within the spherical shells, this effect was ignored, and the albedos of Table IX were employed. To translate β_2 into the critical albedo β_1 at the surface of the metal, all collisions within the shells were assumed to be scattering collisions, the shells were assumed to be slabs, and the following formula was derived:

$$\beta_1 = \beta_0 + \frac{(1 - \beta_0)^2 \beta_2}{1 - \beta_0 \beta_2} \qquad (2)$$

where β_0 is the albedo of the spherical shell. Values of β_0 for the 0.635- and 1.27-cm-thick Plexiglas shells were calculated from spherical extrapolation distances of 2.42 and 2.69 cm to be 0.0531 and 0.1050 at the corresponding critical radii of 8.55 and 8.28 cm. At the 6.44-cm radius of the equivalent spherical unit the respective albedos were calculated from Eq. 18[1] to be 0.0421 and 0.0865. The critical values of β_1 calculated in this manner are given in Table XII together with values of k_{eff} calculated from Eq. 1. Also given are the values of k_{eff} calculated from values of β obtained for the experimentally critical spacings with the interspersed material ignored. It is apparent that the approximate procedure outlined here results in about the same degree of error for the bare and thinly reflected arrays with interspersed material as was obtained without such materials. It is also apparent that ignoring such materials in bare arrays could be dangerous but that it has much less effect in the arrays surrounded by a thick reflector.

CONCLUSIONS

Comparison of calculations made by the method described in Ref. 1 with critical experiments performed with reflected and unreflected arrays of solution units and of metal units shows the method to have reasonable accuracy when consideration is given to its simplicity. The principal trends shown in the comparisons are increases in the values of k_{eff} calculated by Eq. 1 for actually critical arrays as the size of a bare array increases and an increase in k_{eff} for arrays surrounded by a thick reflector over that for unreflected arrays. The similarity of the results obtained with the widely differing types of units to which the calculations were applied gives one confidence in applying the method and in particular, Tables II-V to arrays of units for which data are lacking. Thus, on the basis of Tables VIII and XI, one may conclude (1) that small unreflected arrays calculated to have $k_{\text{eff}} \leq 0.9$ will be subcritical by an adequate margin of safety provided care is taken in the calculations not to underestimate the reactivity of individual units and (2) that large arrays or arrays surrounded by thick reflectors will be safely subcritical at larger values of k_{eff}, chosen by examining Tables VIII and XI, provided the reflector albedos and the area of the reflecting surfaces are chosen in a manner that gives values consistent with those employed in the present paper.

TABLE XII

Effect of Interspersed Material in 8-Unit Arrays of the Smaller Metal Cylinders

				k_{eff}	
Material	Reflector thickness (cm)	Experimental diameter/avg pitch	Critical albedo β	From Eqn. 2	Interspersed material ignored
0.635 cm Thick Plexiglas	0	0.8315	0.3137	0.951	0.889
	1.27	0.7520	0.3337	0.968	0.910
	7.62	0.5406	0.4713	1.090	1.036
	15.24	0.5327	0.5209	1.137	1.083
1.27 cm thick Plexiglas	0	0.7207	0.3062	0.945	0.845
	1.27	0.6485	0.3265	0.962	0.865
	15.24	0.5001	0.5430	1.157	1.053
Pipe	0	0.8888	-	-	0.919
Pipe + Plexiglas	0	0.7844	-	-	0.869